数理統計演習問題

[1] ① $\int_0^2 \frac{3}{4}x^2(2-x)dx = \frac{3}{4}\int_0^2 (2x^2-x^3)dx = \frac{3}{4}\left[\frac{2}{3}x^3-\frac{1}{4}x^4\right]_0^2 = \frac{3}{4}\left(\frac{16}{3}-\frac{1}{4}16\right) = \frac{1}{4}(16-12) = 1.$

② $\int_0^\infty e^{-\theta x}dx = \left[-\frac{1}{\theta}e^{-\theta x}\right]_0^\infty = \frac{1}{\theta}$

③ $\int_0^2\int_0^1 (x+y)^2 dxdy = \int_0^2 \left[\frac{1}{3}(x+y)^3\right]_0^1 dy$

$= \int_0^2 \left(\frac{1}{3}(1+y)^3 - \frac{1}{3}y^3\right)dy$

$= \frac{1}{3}\left[\frac{1}{4}(1+y)^4 - \frac{1}{4}y^4\right]_0^2 = \frac{1}{12}\left(3^4-2^4-1\right) = \frac{64}{12} = \frac{16}{3}$ //

④ $\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \cos x \cos y\, dxdy = \int_0^{\frac{\pi}{2}} \cos y \left[\sin x\right]_0^{\frac{\pi}{2}} dy$

$= \int_0^{\frac{\pi}{2}} \cos y\, dy = 1.$

⑤ $\int_0^1\left(\int_0^{2x}(x+y)^2 dy\right)dx = \int_0^1 \left[\frac{1}{3}(x+y)^3\right]_0^{2x} dx$

$= \int_0^1 \frac{1}{3}\left((3x)^3 - x^3\right)dx$

$= \frac{1}{3}\int_0^1 26x^3 dx$

$= \frac{26}{3}\left[\frac{1}{4}x^4\right]_0^1 = \frac{13}{6}$ //

⑥ $\int_{-\infty}^\infty\int_{-\infty}^\infty (1+x^2+y^2)^{-\frac{3}{2}} dxdy$

$x = r\cos\theta,\ y = r\sin\theta$ とおく

$= \int_0^\infty\int_0^{2\pi} \{1+r^2(\cos^2\theta+\sin^2\theta)\}^{-\frac{3}{2}} \underbrace{|J|}_{= r} d\theta dr$

$= \int_0^\infty \frac{r}{(1+r^2)^{\frac{3}{2}}} \underbrace{\left(\int_0^{2\pi} 1\, d\theta\right)}_{= 2\pi} dr$

$= 2\pi\left[-(1+r^2)^{-\frac{1}{2}}\right]_0^\infty = 2\pi$ //

No. Date

[2] $\int_0^\infty x^A \exp(-Bx)\,dx = \int_0^\infty \left(\frac{y}{B}\right)^A \exp(-y)\,\frac{1}{B}\,dy$

$Bx = y$ とおくと

$dx = \frac{1}{B}dy$ $\quad = B^{-(A+1)} \int_0^\infty y^A \exp(-y)\,dy$

$= B^{-(A+1)}\,\Gamma(A+1)$ //

2 6

[3] $X \sim B(3, \frac{2}{5})$

① $P(X=2) = {}_3C_2\left(\frac{2}{5}\right)^2 \cdot \frac{3}{5} = \frac{36}{125}$ //

$P(X=0) = {}_3C_0\left(\frac{2}{5}\right)^0\left(\frac{3}{5}\right)^3 = \frac{27}{125}$

$P(X=1) = {}_3C_1\left(\frac{2}{5}\right)\left(\frac{3}{5}\right)^2 = \frac{54}{125}$

$P(X=3) = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$

② $X \sim B(3, \frac{2}{5})$.

③ $E(X) = \frac{6}{5}$

④ $V(X) = 3 \cdot \frac{2}{5} \cdot \frac{3}{5} = \frac{18}{25}$ //

⑤ $D(X) = \sqrt{\frac{18}{25}} = \frac{3\sqrt{2}}{5}$

[4] $f(x) = \theta e^{-\theta x}$

① $E(X) = \int_0^\infty x \cdot \theta e^{-\theta x}\,dx = \theta \int_0^\infty x\, e^{-\theta x}\,dx$ [2]において $A=1$, $B=\theta$ とおくと

$= \theta \quad \theta^{-2}\, \underbrace{\Gamma(2)}_{=1}$

$= \frac{1}{\theta}$ //

② $E(X^2) = \int_0^\infty x^2\, \theta e^{-\theta x}\,dx$

$= \theta \int_0^\infty x^2 e^{-\theta x}$ [2]において $A=2$, $B=\theta$ とおくと

$= \theta \cdot \theta^{-3}\,\Gamma(3)$

$= \theta^{-2}\, 2\, \underbrace{\Gamma(2)}_{=1}$

$\therefore V(X) = E(X^2) - E(X)^2 = \frac{2}{\theta^2} - \left(\frac{1}{\theta}\right)^2 = \frac{1}{\theta^2}$ //