Σ は知っている

Date . . No. 1

・$x_1, \ldots, x_N, y_1, \ldots, y_N$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \quad \overline{x^2} = \frac{1}{N}\sum_{i=1}^{N} x_i^2 \quad \overline{xy} = \frac{1}{N}\sum_i x_i y_i$$

$(-\infty, \infty) = \mathbb{R}$：実数全体の集合

$\mathbb{R}^N$：N次元

$$\underbrace{(-\infty,\infty)\times\cdots\times(-\infty,\infty)}_{N \text{コ}}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = \int_{\mathbb{R}^2} f(x,y)\,dx\,dy$$

行列は A or a と表わす

[行列の和,差]：
P56-59.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$$

[積]
P57-60

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 1\cdot5+2\cdot7 & 1\cdot6+2\cdot8 \\ 3\cdot5+4\cdot7 & 3\cdot6+4\cdot8 \end{pmatrix}$$

$$= \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$$

P60 ・(6.51) (6.52).

[スカラー倍]
実数倍
P60

$$a\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} a & 2a \\ 3a & 4a \end{pmatrix}$$

性質 A, B, C, D 行列

① ・一般的に $AB \neq BA$

・$aBC = B(aC)$ P62 Fact 6.2

・$A(B+C) = AB+AC$ P201 Fact 22.4

$(B+C)A = BA+CA$

・$(A+B)(C+D) = AC+AD+BC+BD$ P202 Fact 22.6

・$ABCD = A(B(CD))$ P206

[単位行列] $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\underset{(=I_N)}{I} = \underbrace{\begin{pmatrix} 1 & & O \\ & \ddots & \\ O & & 1 \end{pmatrix}}_{N}\Big\} N$

・$AI = IA = A$

No. Date . .

P212

[逆行列]: 正方行列A

$AB = I$ かつ $BA = I$

の時 BはAの逆行列と言い A^{-1}と書く

i.e., $AA^{-1} = I$ かつ $A^{-1}A = I$.

P215 Fact 22.15

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ の時 $A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$

3×3行列の逆行列 → 公式, 掃き出し法

性質 $(AB)^{-1} = B^{-1}A^{-1}$ $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

[ベクトル] $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_N \end{pmatrix}$

$A = \begin{pmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & & \vdots \\ a_{N1} & \cdots & a_{NM} \end{pmatrix} = (\mathbf{a}_1, \mathbf{a}_2, \cdots \mathbf{a}_M).$

$\mathbf{a}_1 = \begin{pmatrix} a_{11} \\ \vdots \\ a_{N1} \end{pmatrix}$ $\mathbf{a}_2 = \begin{pmatrix} a_{12} \\ \vdots \\ a_{N2} \end{pmatrix}$ $\cdots$ $\mathbf{a}_M = \begin{pmatrix} a_{1M} \\ \vdots \\ a_{NM} \end{pmatrix}$

[行列式] $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ をAの行列式と言う

$= ad - bc$

[trace]

$$A=\begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix},\quad B=\begin{pmatrix} b_{11} & \cdots & b_{1N} \\ \vdots & & \vdots \\ b_{N1} & \cdots & b_{NN} \end{pmatrix}$$

$\mathrm{tr}(A) = a_{11} + a_{22} + \cdots + a_{NN}$ と定義.

- $\mathrm{tr}(A+B) = \mathrm{tr}(A) + \mathrm{tr}(B)$, • $\mathrm{tr}(A-B) = \mathrm{tr}(A) - \mathrm{tr}(B)$
- $\mathrm{tr}(I_N) = N$.

Fact 22.34

$$A=\begin{pmatrix} a_{11} & \cdots & a_{1K} \\ & & \vdots \\ a_{N1} & \cdots & a_{NK} \end{pmatrix},\quad B=\begin{pmatrix} b_{11} & \cdots & b_{1N} \\ \vdots & & \vdots \\ b_{K1} & \cdots & b_{KN} \end{pmatrix}$$

$\mathrm{tr}(AB) = \mathrm{tr}(BA)$.

P232.

ex. $A=\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$ $B=\begin{pmatrix} -1 & 0 & 1 \\ 0 & 2 & -3 \end{pmatrix}$

$$AB=\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}\begin{pmatrix} -1 & 0 & 1 \\ 0 & 2 & -3 \end{pmatrix}=\begin{pmatrix} -1 & 4 & -5 \\ -3 & 8 & -9 \\ -5 & 12 & -13 \end{pmatrix} \qquad \mathrm{tr}(AB) = -1+8-13 = -6$$

$$BA=\begin{pmatrix} -1 & 0 & 1 \\ 0 & 2 & -3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}=\begin{pmatrix} 4 & 4 \\ -9 & -10 \end{pmatrix} \qquad \mathrm{tr}(BA) = -6$$

No. Date . .

[転置行列]

$A=\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ とする時 $A'=\begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix}$ とAの転置行列と言う.

P200 性質

Fact

22.1° $(A+B)'=A'+B'$

22.2° $(A-B)'=A'-B'$

22.3° $(AB)'=B'A'$ $(ABC)'=C'B'A'$

$(A')'=A$

[対称行列] 正方行列Aにおいて

$A'=A$ の時, Aを対称行列と言う

ex

$A=\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ $A'=\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ $A=A'$.

$B=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $B'=\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ $B\neq B'$

Fact 22.17

A:対称行列. の時 A^{-1} も対称行列.

$A'=A \Rightarrow (A^{-1})'=A^{-1}$

$I=I'=(AA^{-1})'=(A^{-1})'A'=(A^{-1})'A$ $\therefore I=(A^{-1})'A$ 右からA^{-1}をかける

$A^{-1}=(A^{-1})'$ □

$$A=\begin{pmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & & \vdots \\ a_{N1} & \cdots & a_{NM} \end{pmatrix} = \begin{pmatrix} \mathbf{a}_1' \\ \mathbf{a}_2' \\ \vdots \\ \mathbf{a}_N' \end{pmatrix}.$$

$$\mathbf{a}_\ell' = (a_{\ell 1} \cdots a_{\ell M}) \qquad \mathbf{a}_\ell = \begin{pmatrix} a_{\ell 1} \\ \vdots \\ a_{\ell M} \end{pmatrix}$$

ベクトル $\mathbf{a}_1, \ldots, \mathbf{a}_N$ が独立 $\Leftrightarrow$ $c_1\mathbf{a}_1 + c_2\mathbf{a}_2 + \cdots + c_N\mathbf{a}_N = \mathbf{0}$
$\Rightarrow c_1 = c_2 = \cdots = c_N = 0$

$\mathbf{a}_1, \ldots, \mathbf{a}_N$ が従属 $\Leftrightarrow$ $c_1\mathbf{a}_1 + c_2\mathbf{a}_2 + \cdots + c_N\mathbf{a}_N = \mathbf{0}$
かつ. $c_1, \ldots, c_N$ が全て0とはならない

[rank]. $\mathrm{rank}(A)$: $A=\begin{pmatrix} \mathbf{a}_1' \\ \vdots \\ \mathbf{a}_N' \end{pmatrix}$ において独立なベクトルの最大数.

ex $A=\begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix}$ $\mathbf{a}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\mathrm{rank}(A)=1$.

$\mathbf{a}_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} = 2\mathbf{a}_1$

$\mathbf{a}_3 = \begin{pmatrix} 3 \\ 6 \end{pmatrix} = 2\mathbf{a}_1$

P230.

Mはベキ等行列 $\overset{\mathrm{def}}{\Leftrightarrow}$ $M^2 = M$.

P234 Fact 22.36

$$\boxed{\begin{array}{l} M\text{は対称なベキ等行列.} \\ \mathrm{rank}(M) = \mathrm{tr}(M). \end{array}}$$

P235 Fact 22.37. $X = \begin{pmatrix} x_{11} & \cdots & x_{1K} \\ & & \vdots \\ x_{N1} & \cdots & x_{NK} \end{pmatrix}$ とする.

$\mathrm{rank}(I - X(X'X)^{-1}X') = N - K$.

No. Date . .

$M = I - X(X'X)^{-1}X'$ は対称なべキ等行列

$\mathrm{rank}(M) = \mathrm{tr}(I_N - X(X'X)^{-1}X')$ ∵ Fact 22.36

$= \mathrm{tr}(I_N) - \mathrm{tr}(X(X'X)^{-1}X')$

$= N - \mathrm{tr}(\underbrace{(X'X)^{-1}X'X}_{= I_K})$

$= N - K.$ □

偏微分

$f(\beta_1, \beta_2, \dots, \beta_k) = f(\beta)$. $\quad \beta = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_k \end{pmatrix}$

P103

$$\frac{\partial f(\beta)}{\partial \beta} \equiv \begin{pmatrix} \frac{\partial f(\beta)}{\partial \beta_1} \\ \vdots \\ \frac{\partial f(\beta)}{\partial \beta_k} \end{pmatrix}$$

$$\frac{\partial f(\beta)}{\partial \beta'} = \left(\frac{\partial f(\beta)}{\partial \beta_1} \cdots \frac{\partial f(\beta)}{\partial \beta_k} \right)$$

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1k} \\ \vdots & & \vdots \\ a_{k1} & \cdots & a_{kk} \end{pmatrix} = \begin{pmatrix} a_1' \\ \vdots \\ a_k' \end{pmatrix} = (\bar{a}_1 \cdots \bar{a}_k) \quad \leftarrow \text{ex.}$$

・ $f(\beta) = \beta' A \beta$

$$= \sum_i \sum_j a_{ij} \beta_i \beta_j \qquad \leftarrow \text{k=2の場合のex.}$$

$$= \sum_{i=1}^{k} a_{ii} \beta_i^2 + \sum_{i \neq j} \sum a_{ij} \beta_i \beta_j$$

$$\frac{\partial f(\beta)}{\partial \beta_\ell} = \frac{\partial}{\partial \beta_\ell} \Big(a_{\ell\ell} \beta_\ell^2 + a_{\ell 1} \beta_\ell \beta_1 + a_{\ell 2} \beta_\ell \beta_2 + \cdots + a_{\ell k} \beta_\ell \beta_k$$
$$+ a_{1\ell} \beta_1 \beta_\ell + a_{2\ell} \beta_2 \beta_\ell + \cdots + a_{k\ell} \beta_k \beta_\ell + \text{定数} \Big)$$

$$= 2a_{\ell\ell} \beta_\ell + a_{\ell 1} \beta_1 + a_{\ell 2} \beta_2 + \cdots + a_{\ell k} \beta_k$$
$$+ a_{1\ell} \beta_1 + \cdots \cdots + a_{k\ell} \beta_k$$

$$= \bar{a}_\ell \beta + a_\ell' \beta$$

No. Date . .

性質

$$\frac{\partial \beta' A \beta}{\partial \beta} = \begin{cases} (A+A')\beta & \text{P208 Fact 22.11} \\ 2A\beta & (\text{Aが対称行列}) \quad \text{Fact 22.12} \end{cases}$$

おまけ $\displaystyle \frac{\partial \beta' A \beta}{\partial \beta \partial \beta^T} = \begin{cases} A+A' \\ 2A \end{cases}$

$\mathbf{a} = \begin{pmatrix} a_1 \\ \vdots \\ a_k \end{pmatrix}$ とおく時

$$\frac{\partial \mathbf{a}'\beta}{\partial \beta} = \frac{\partial \beta' \mathbf{a}}{\partial \beta} = \mathbf{a} \qquad \text{P207 Fact 22.9, 22.10}$$

PROOF

$$\frac{\partial f(\beta)}{\partial \beta} = \begin{pmatrix} \frac{\partial f(\beta)}{\partial \beta_1} \\ \vdots \\ \frac{\partial f(\beta)}{\partial \beta_k} \end{pmatrix} = \begin{pmatrix} \mathbf{a}_1' \beta \\ \vdots \\ \mathbf{a}_k' \beta \end{pmatrix} + \begin{pmatrix} \bar{\mathbf{a}}_1 \beta \\ \vdots \\ \bar{\mathbf{a}}_k \beta \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{a}_1' \\ \vdots \\ \mathbf{a}_k' \end{pmatrix} \beta + \begin{pmatrix} \bar{\mathbf{a}}_1 \\ \vdots \\ \bar{\mathbf{a}}_k \end{pmatrix} \beta$$

$$= (\mathbf{a}_1 \cdots \mathbf{a}_k)' \beta + \begin{pmatrix} \bar{\mathbf{a}}_1 \\ \vdots \\ \bar{\mathbf{a}}_k \end{pmatrix} \beta$$

$$= A'\beta + A\beta$$

$$\mathbf{a}'\beta = a_1\beta_1 + \cdots + a_k\beta_k$$

$$\frac{\partial}{\partial \beta_\ell} \mathbf{a}'\beta = a_\ell$$

$$\frac{\partial \mathbf{a}'\beta}{\partial \beta} = \begin{pmatrix} a_1 \\ \vdots \\ a_k \end{pmatrix} = \mathbf{a}.$$

0 準備

[確率変数]

X は pdf $f(x)$ を持つ

$$P(X \in A) = \int_A f(x)\,dx \qquad \text{ex. } A = (a, b)$$

$$P(X \in A) = P(a < X < b) = \int_a^b f(x)\,dx$$

P27-28

期待値 $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx \quad (\equiv \mu)$.

$$E(g(X)) = \int_{-\infty}^{\infty} g(x) f(x)\,dx.$$

分散 $V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx = E[(X-\mu)^2]$.

性質 a, b：定数 (P43-48).

$E(aX+b) = aE(X) + b$

$V(aX+b) = a^2 V(X)$

[同時確率分布]

$\mathbb{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ は 同時 pdf $f(x_1, x_2)$ を持つ.

$$E(X_1) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 f(x_1, x_2)\,dx_1 dx_2 = \int_{\mathbb{R}^2} x_1 f(x_1, x_2)\,dx_1 dx_2 \quad (\equiv \mu_1).$$

$$E(X_2) = \int_{\mathbb{R}^2} x_2 f(x_1, x_2)\,dx_1 dx_2 \quad (\equiv \mu_2)$$

$$E(g(X_1, X_2)) = \int_{\mathbb{R}^2} g(x_1, x_2) f(x_1, x_2)\,dx_1 dx_2$$

$$Cov(X_1, X_2) = \int_{\mathbb{R}^2} (x_1 - \mu_1)(x_2 - \mu_2) f(x_1, x_2)\,dx_1 dx_2$$

$$\left(= E((X_1 - \mu_1)(X_2 - \mu_2)).\right)$$

No. Date . .

小性質

$$Cov(aX_1, bX_2) = ab\,Cov(X_1, X_2).$$

PROOF. $Cov(aX_1, bX_2) = E((aX_1 - E(aX_1))(bX_2 - E(bX_2)))$

$= E(a(X_1 - \mu_1)b(X_2 - \mu_2))$

$= ab\,Cov(X_1, X_2)$

[確率ベクトル] P64-65

$$\mathbb{X} = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_k \end{pmatrix}$$

$$E(\mathbb{X}) \equiv \begin{pmatrix} E(X_1) \\ E(X_2) \\ \vdots \\ E(X_k) \end{pmatrix} \equiv \boldsymbol{\mu}$$: 平均ベクトル

P73.

$V(\mathbb{X}) \equiv E((\mathbb{X} - \boldsymbol{\mu})(\mathbb{X} - \boldsymbol{\mu})') \equiv \Sigma$: (分散)共分散行列.

$$= E\left[\begin{pmatrix} X_1 - \mu_1 \\ \vdots \\ X_k - \mu_k \end{pmatrix} (X_1 - \mu_1 \ \cdots \ X_k - \mu_k)\right]$$

$$= E\left[\begin{pmatrix} (X_1-\mu_1)^2 & \cdots & (X_1-\mu_1)(X_k-\mu_k) \\ (X_1-\mu_1)(X_2-\mu_2) & & \vdots \\ \vdots & & \\ (X_1-\mu_1)(X_k-\mu_k) & \cdots & (X_k-\mu_k)^2 \end{pmatrix}\right]$$

$$= \begin{pmatrix} V(X_1) & \cdots & & Cov(X_1, X_k) \\ Cov(X_1, X_2) & V(X_2) & & \vdots \\ \vdots & & \ddots & \vdots \\ Cov(X_1, X_k) & \cdots & & V(X_k) \end{pmatrix}$$

性質

A: M×K行列 $\mathbb{b} \in \mathbb{R}^k$.

$$
\overset{k}{\overbrace{\begin{pmatrix} a_{11} & \cdots & a_{1k} \\ \vdots & & \vdots \\ a_{M1} & \cdots & a_{Mk} \end{pmatrix}}} \Big\} M
$$

- $E(A\mathbb{X}+\mathbb{b}) = AE(\mathbb{X})+\mathbb{b} \quad (\equiv A\mu+\mathbb{b})$
- $E(A\mathbb{X}B) = AE(\mathbb{X})B$ 　行列のかける順番は変わらない

行列

- $V(A\mathbb{X}) = AV(\mathbb{X})A'$

ex. $\mathbb{X}=\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$ $A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$ $\mathbb{b}=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

$$
A\mathbb{X}+\mathbb{b} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_{11}X_1+a_{12}X_2+b_1 \\ a_{21}X_1+a_{22}X_2+b_2 \\ a_{31}X_1+a_{32}X_2+b_3 \end{pmatrix}
$$

$$
E(A\mathbb{X}+\mathbb{b}) = \begin{pmatrix} E(a_{11}X_1+a_{12}X_2+b_1) \\ E(a_{21}X_1+a_{22}X_2+b_2) \\ E(a_{31}X_1+a_{32}X_2+b_3) \end{pmatrix}
$$

$$
= \begin{pmatrix} a_{11}E(X_1)+a_{12}E(X_2)+b_1 \\ a_{21}E(X_1)+a_{22}E(X_2)+b_2 \\ a_{31}E(X_1)+a_{32}E(X_2)+b_3 \end{pmatrix}
$$

$$
= \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\begin{pmatrix} E(X_1) \\ E(X_2) \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}
$$

$$
= AE(\mathbb{X})+\mathbb{b}.
$$

No. Date . .

統計的独立性

$\mathbb{X}_1 = \begin{pmatrix} X_1 \\ \vdots \\ X_\ell \end{pmatrix}$ と $\mathbb{X}_2 = \begin{pmatrix} X_{\ell+1} \\ \vdots \\ X_k \end{pmatrix}$ は独立. ← $\mathbb{X}_1 \perp\!\!\!\perp \mathbb{X}_2$

$\Leftrightarrow \underset{\uparrow}{f(x_1, x_2)} = \underset{\uparrow}{f(x_1)}\, \underset{\uparrow}{f(x_2)}$

$\mathbb{X}_1$ と $\mathbb{X}_2$ の同時pdf　$\mathbb{X}_1$ のpdf　$\mathbb{X}_2$ のpdf

◎ $f(x_i)$: X_i の pdf. $(i=1,2,\ldots,N)$

$X_1, X_2, \ldots, X_N \overset{iid}{\sim}$　　iid = idependently identically distributed

$$f(x_1, x_2, \ldots, x_N) = f(x_1) f(x_2) \cdots f(x_N)$$
$$= \prod_{i=1}^{N} f(x_i)$$

P251

[カイ2乗分布]: 確率変数 X の pdf が

$$f(x)=\begin{cases}\dfrac{1}{2\Gamma(\frac{r}{2})}\left(\dfrac{x}{2}\right)^{\frac{r}{2}-1}e^{-\frac{x}{2}} & (x>0)\\ 0 & (x\leq 0)\end{cases}$$

の時, Xは自由度rのχ^2分布に従うと言う

$$X\sim\chi^2(r)$$

$\Gamma(p)=\int_0^\infty t^{p-1}e^{-t}dt$ はガンマ関数

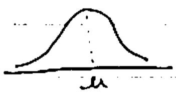
pは整数 $\Gamma(p)=(p-1)!$　　$\Gamma(\frac{1}{2})=\sqrt{\pi}$.

・$E(X)=r$　　・$V(X)=2r$.

[多変量正規分布]

P136 $\varepsilon \sim N(\mu, \sigma^2)$

$\Leftrightarrow$ ε の pdf. $f(\varepsilon \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(\varepsilon-\mu)^2}{2\sigma^2}\right\}$.



$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{pmatrix}$$

$\boldsymbol{\varepsilon}$ は平均 $\boldsymbol{\mu}$, 共分散行列 Σ の N 変量正規分布 $\boldsymbol{\varepsilon} \sim N(\boldsymbol{\mu}, \Sigma)$.

$\Leftrightarrow$ $\boldsymbol{\varepsilon}$ の pdf.

$$f(\boldsymbol{\varepsilon} \mid \boldsymbol{\mu}, \Sigma) = \frac{1}{\sqrt{(2\pi)^N \underbrace{|\Sigma|}_{\text{行列式}}}} \exp\left\{-\frac{1}{2}(\boldsymbol{\varepsilon}-\boldsymbol{\mu})' \Sigma^{-1} (\boldsymbol{\varepsilon}-\boldsymbol{\mu})\right\}.$$

$\exp(\square) = e^{\square}$

$N(\mathbf{0}, I_N)$: N 変量標準正規分布

$$f(\boldsymbol{\varepsilon} \mid \mathbf{0}, I_N) = \frac{1}{\sqrt{(2\pi)^N}} \exp\left\{-\frac{1}{2}\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon}\right\}$$

$\boldsymbol{\varepsilon}' I_N \boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}'\boldsymbol{\varepsilon}$.

$\varepsilon_1, \dots, \varepsilon_N \overset{iid}{\sim} N(0,1)$.

iid 互いに独立に同一分布に従う

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_N \end{pmatrix}$$

$\boldsymbol{\varepsilon}$ の pdf

$$\begin{aligned} f(\boldsymbol{\varepsilon}) &= f(\varepsilon_1, \dots, \varepsilon_N) \\ &= f(\varepsilon_1) \cdots f(\varepsilon_N) \\ &= \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x_i^2} \\ &= \frac{1}{\sqrt{(2\pi)^N}} e^{-\frac{1}{2}\sum \varepsilon_i^2} \\ &= \frac{1}{\sqrt{(2\pi)^N}} e^{-\frac{1}{2}\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon}} \leftarrow N(\mathbf{0}, I_N). \end{aligned}$$

No. Date . .

Fact 23.2 (P238)

$\mathbb{X} \sim N(\mu, \Sigma)$.

・$E(\mathbb{X}) = \mu$
・$V(\mathbb{X}) = \Sigma$ } ← 証明略. (佐和 P45)

・$\mathbb{Y} = A\mathbb{X} + \mathbb{b}$. とおくと $\mathbb{Y} \sim N(A\mu + \mathbb{b}, A\Sigma A')$.

Proof. $\mathbb{X} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \sim N(\mu, \Sigma)$ P238 定理23.1. 類

$\Rightarrow$ $t'\mathbb{X} + t_0 = t_0 + t_1 x_1 + \cdots + t_n x_n$ は正規分布に従う. ← 積率母関数を使う.

$$E(\mathbb{Y}) = E(A\mathbb{X} + \mathbb{b}) = AE(\mathbb{X}) + \mathbb{b} = A\mu + \mathbb{b}.$$

$$\begin{aligned} V(\mathbb{Y}) &= E[(A\mathbb{X} + \mathbb{b} - A\mu - \mathbb{b})(A\mathbb{X} + \mathbb{b} - A\mu - \mathbb{b})'] \\ &= E[A(\mathbb{X} - \mu)(\mathbb{X} - \mu)'A'] \\ &= A\,E[(\mathbb{X} - \mu)(\mathbb{X} - \mu)']A' \\ &= A\Sigma A' \end{aligned}$$