The Economic Journal of Takasaki City University of Economics vol.48 No.3 2006 pp.117～133

Dimensions, Stocks and Flows

Kei NAKAJI[1]

Abstract

This paper aims to clarify the conceptions of dimensions, stocks and flows, and their role in economic theory. Surely, almost every textbook in economics stresses the importance of them. Nonetheless, it is undeniable that some standard economic models ignore them and violate the rule as to time and dimensions.

First, we define main concepts adopted in this paper; time dimensions, stocks and flows, and unit of measurement of economic quantities. Second, we try to apply them to some of familiar economic theory, namely the wages-fund theory, theory of consumer behavior, production functions and distribution. Finally, we analyze IS-LM.

We ascertain the defect of some of modern economic theories in the light of dimension and time conceptions and see that, even if they contain no errors mathematically, they often mistake formal consistency for the truth in economic theory, and in some cases, they commit self-contradiction.

＊JEL Classifications: A10 B12 B20 B41

【Contents】

1. Introduction

The title of this paper is quite the same with that of the short appendix I in Currie and Steedman [5]. No one can deny that Currie and Steedman examine the means to handle with "time" concept in economic theory through out their insightful and suggestive book in terms of history of modern economics. They trace carefully traits and developments of time conceptions of great economists, *i.e.*, A. Marshall, M.E.L. Walras, E. Lindahl, J.R. Hicks, G.L.S. Shackle, and L.M. Lachman [2]. To read their work carefully, everyone would accept their conclusion that "no unique temporal framework has secured universal acceptance amongst economists" ([5] p.240).

They argue dimensions, stocks and flows in the Appendix I. It is unfortunate that no clear vision or implication of them to economic theory is presented. This paper aims to clarify the concepts of dimensions, stocks and flows in the context of modern economics that Currie and Steedman did not try to examine in detail.

Surely, many standard textbooks in economics stressed on the importance and significance of distinction between stocks and flows. Nonetheless, the consideration to it is not necessarily enough, especially in the discussion on the basic concepts of economics.

2. Definitions and Notations

2.1. Definitions of Dimensions, Stocks and Flows

2.1.1. Dimensions, or unit of measurement

All economic quantities must have time dimension T as well as any unit or dimension of measurement. That is to say, an economic quantity in general requires two properties, time and accounting unit. As to time dimension, we discuss in the next section, so begin with units of account or measurement.

For example, a person's cash balance (B) is expressed by, *say, Yen.* We may notate this example as following.

$$\dim(B) = [yen] \tag{2.1.1}$$

Needless to say, this means a cash balance B belongs to a nominal or monetary unit. If needed, a nominal quantity (m) in general is expressed as follows.

$$\dim(m) = [nom] \tag{2.1.2}$$

Similarly, a physical or real quantity(x) is

$$\dim(x) = [real] \tag{2.1.3}$$

Note that each real quantity has different real dimension, *e.g.,* beef is measured by *kg.,* beer by *litter,* labor force by *the numbers of people.* Say,

$$\dim(x) = [real\,(kg)] \; or \; = [real\,(l)] \; etc.$$

To combine these, we may notate price of any commodity (p).

$$\dim(p) = \left[nom \times \{real\}^{-1}\right] = \left[\frac{nom}{real}\right] \tag{2.1.4}$$

This formal expression tells us that price of any commodity (p) is defined as the quantity of money to buy or command one unit of any kind commodity (*e.g.* 1kg. of beef), or exchange rate of a commodity for money expenditure [3].

When and only if the same unit measures two economic quantities, the *ratio* of them is called a *pure number. I.e.,*

$$\dim\left(\frac{x_1}{x_2}\right) = \left[\frac{real}{real}\right] = [pure] \tag{2.1.5}$$

being both x_1 and x_2 the same real quantity, thus the number of apples in the hands of two different individuals now.

Or,

$$\dim\left(\frac{y_1}{y_2}\right) = \left[\frac{nom}{nom}\right] = [pure] \tag{2.1.6}$$

being both y_1 and y_2 the same nominal quantity, for example, the pecuniary income of 1st person and that of 2nd person in this year.

2.1.2. Time in Economics (I); Stock Variables

Stock variables are defined as the quantities exist in any instant τ_0. Demand for any commodity (x_1) is clearly a stock quantity changing time to time, and using formulation (2.1.3), hence we may express as below.

$$\dim(x_1) = [real,\ T \in \tau_0] \tag{2.1.7}$$

It must be noted that as long as different two stocks, x_1 and x_2, belong to the same real accounting unit *and* the same point of time, $x_1 \pm x_2$, have make a sense in economics. Thus, we cannot add one dozen of pencils with an eraser in our hand now[4], nor subtract an apple today from two apples one year hence[5], even if apples today and one year hence are measured by same real unit (say *g.*)[6]. This holds true money flows during two separate periods, of course.

2.1.3. Time in Economics (II); Flow Variables

A flow variable is an economic quantity defined only during any time interval. Time interval is, of course, specified by not only its length but starting point of time. For example, we write any person's pecuniary income y during this year t_0 that starts at τ_0 and ends at τ_1.

$$\dim(y) = [nom,\ T \in t_0(\tau_0 \to \tau_1)] \tag{2.1.8}$$

As the case of stocks, when we make summing or deducting manipulation of two flows or more, they must be belong to the strictly same time interval as well as have the same accounting unit. As we shall discuss later, if they start at different points of time and/or they are belong to the different time intervals, we must introduce discount rate to handle them as if they exist in the exactly same period, or to capitalize, that is, to follow a due procedure to reduce flows to stocks (*vide* section 2.3).

2.2. Formal Relationship of Stock and Flow

By far, we discuss stocks and flows separately. To make our idea clearer, we investigate into their relationship.

First, we assume a stock(S) in this moment consists of two flows, that is inflow (I) and outflow (O), and an old stock already established (S')[7] .Namely

$$S = S' + I - O \tag{2.2.1}$$

Here, we note any unit U measures all quantities in question, therefore,

$$\begin{aligned}
\dim(S) &= [real(U),\ T \in \tau_1] \\
\dim(S') &= [real(U),\ T \in \tau_0] \\
\dim(I) &= [real(U),\ T \in t_0(\tau_0 \to \tau_1)] \\
\dim(O) &= [real(U),\ T \in t_0(\tau_0 \to \tau_1)]
\end{aligned} \tag{2.2.2}$$

We may rewrite the formula above as follows.

$$S - S' = I - O \tag{2.2.3}$$

Here it seems that we may violate the rule that it is impossible to subtract a stock quantity, *S'*, from a stock of different time dimension, *S*. We define, however, stocks existing at the different points of time in the connection with flows of which time dimension covers those of stocks, so that (2.2.3) pictures, say, a reservoir to which water is pouring and of which water is flowing out, or compares to a person's cash balance in a bank into and from which money flows day by day.

We can divide this equation by time span t_0 to define the ***average*** rates of change of stock and flow.

$$\frac{S - S'}{t_0} = \frac{I}{t_0} - \frac{O}{t_0} \quad or \quad \hat{S} = \hat{I} - \hat{O} \tag{2.2.4}$$

If t_0 approaches the infinitesimal dt (τ_1 approaches τ_0), (2.2.4) becomes,

$$\frac{dS}{dt} = \frac{dI}{dt} - \frac{dO}{dt} \quad or \quad \dot{S} = \dot{I} - \dot{O} \tag{2.2.5}$$

We may say the ***marginal*** rate of change of a stock equals marginal rate of flows. Alternatively, the rate of change of stock is defined by the rates of change of inflow and outflow. Time dimensions are

$$\begin{aligned} \mathbf{dim}\,(\dot{S}) &= \left[real\,(U),\, T \in \tau_0\right] \\ \mathbf{dim}\,(\dot{I}) &= \left[real\,(U),\, T \in \tau_0\right] \\ \mathbf{dim}\,(\dot{O}) &= \left[real\,(U),\, T \in \tau_0\right] \end{aligned} \tag{2.2.6}$$

Clearly, there is no inconsistency with (2.2.5). If we divide (2.2.4) by *S*, then we get

$$\frac{\hat{S}}{S} = \frac{\hat{I}}{S} - \frac{\hat{O}}{S} \quad or \quad \frac{S - S'}{S \cdot t_0} = \frac{I}{S \cdot t_0} - \frac{O}{S \cdot t_0} \tag{2.2.7}$$

Similarly, from (2.2.5)

$$\frac{\dot{S}}{S} = \frac{\dot{I}}{S} - \frac{\dot{O}}{S} \quad or \quad \frac{dS}{S \cdot dt} = \frac{dI}{S \cdot dt} - \frac{dO}{S \cdot dt} \tag{2.2.8}$$

The rate of change of a stock variable, average or marginal, equals the difference of the rates of change of an inflow and outflow. It is noteworthy that the formula (2.2.8) is of no time dimension. In other words, infinitesimal calculus eliminates not merely nonlinear character of a changing economic variable but time dimension, to make stock-flow relationship meaningless. For example,

$$\dim\left(\frac{\dot{S}}{S}\right)=\left[\frac{real\,(U),\ T\in\tau_0}{real\,(U),\ T\in\tau_0}\right]=[pure] \tag{2.2.9}$$

2.3. Discount rate and Capitalization

As well known, when \$100 bears interest \$3 at the maturity date (the amount then is \$103) or \$3 perpetual annuity at the first day in every year [8], we say the rate of interest is 3%.

$$1+r=\frac{103}{100},\quad or\quad r=\frac{3}{100} \tag{2.3.1}$$ [9]

We ascertain the dimension of the rate of interest of price sense, assuming $\boldsymbol{S}$ and $\boldsymbol{R}$ remain constant and no depreciation or risk.

$$r=\frac{R/t}{S} \tag{2.3.2}$$

Here, we notice that S is a stock that exists at every moment.

$$\dim(S)=[nom,\ T\in\tau_n]^{for\ \forall n} \tag{2.3.3}$$

Accordingly,

$$\dim(r)=\frac{[nom,\ T\in\tau_1]}{[nom,\ T\in\tau_0][T\in t(\tau_0\rightarrow\tau_1)]}=\frac{[nom,\ T\in\tau_1]}{[nom,\ T\in\tau_1][T\in t(\tau_0\rightarrow\tau_1)]}$$
$$=\frac{[pure]}{[T\in t(\tau_0\rightarrow\tau_1)]} \tag{2.3.4}$$

That is to say, the dimension of the rate of interest is not of pure number as Allen asserted, but of reciprocal of time duration[10]. In addition, the dimension of reciprocal of the rate of interest is simply time interval, say, one year. When we say a person's real estate is worth "years' purchase," we keep time dimension of the rate of interest in our mind[11]. As an illustration, if $r=4\%$ and a person's property worth \$2,500, then this property bears \$100 per year, so that years' purchase is worth 25 year-income.

$$\frac{2,500}{100}=25=\frac{1}{0.04}=\frac{1}{r} \tag{2.3.5}$$

Hence,

$$\dim\left(\frac{1}{r}\right)=[T\in t(\tau_0\rightarrow\tau_1)] \tag{2.3.6}$$

The rate of interest may be defined in any time span, one year, a month, a day, a week, and if need, one moment. Thus, we should reject Allen's assertion that the rate of interest is of no time dimension, a pure number. We will get the instantaneous rate of interest (i) in terms of the infinitesimal dt.

$$1 + r = \lim_{t \to \infty}\left(1 + \frac{r'}{t}\right)^{t} = e^{i} \tag{2.3.7}$$

Here, r' is a partial payment during any sub-interval 1/t, say a half of a year, one-twelfth of a year, and so on. Hence,

$$i = \ln\left(1 + r\right) \tag{2.3.8}$$

We use two types of the rate of interest to get present discount value (PDV); the former for discrete payments $R^{D}{}_{t}$, or intertemporal stock variables and the latter for continuous payments $R^{c}{}_{t}$, or intertemporal flow variables. So to speak, we treat the rates of interest as discount factor. Assuming both perpetual annuity and the rates of interest remain constant, the simplest forms of PDV are

$$PDV = \sum_{t=1}^{\infty}\frac{R^{D}{}_{t}}{\left(1 + r\right)^{t}} = \frac{R}{r} \tag{2.3.9}$$

$$or \quad = \int_{0}^{\infty} R^{c}{}_{t} \cdot \exp\left(-i \cdot t\right) dt = \frac{R}{i} \tag{2.3.10}$$

Thus, we can capitalize any stock variables existing at any future points of time as well as flows in any future intervals, to take them back to "*Dasein.*" Though the procedure of capitalization is quite important, many standard economic theories including "dynamic theory" neglect them.

To verify the time dimension of PDV, assuming discrete cash "flows" existing at the starting point of time of all periods are constant,

$$\dim\left(PDV\right) = \left[\frac{\dfrac{\left[nom,\, T \in \tau_{0}\right]}{\left[T \in t\left(\tau_{0} \rightarrow \tau_{1}\right)\right]}}{\dfrac{\left[pure\right]}{\left[T \in t\left(\tau_{0} \rightarrow \tau_{1}\right)\right]}}\right] = \left[nom,\, T \in \tau_{0}\right] \tag{2.3.11}$$

Namely, any economic quantity in the future is reduced a present stock by means of discount rate. For example, when $r = 3\%$, perpetual annuity or consol dividend \$3 paid at the first day of each year is equivalent to \$100 as a stock in our hand now.

It must be noticed that without a rate of interest or discount and/or stock-flow relationship, we cannot operate mathematically different stocks and/or different flows each other.

3. Applications and Verifications

3.1. Classical Wages-fund Theory

J.S. Mill recanted, as well known, his wages-fund theory in front of W. Thornton's critique ([20]) that there was no determining factor of the amount of wages-fund *per se*[12].

Mill exposures the wages-fund theory as such.

> "Wages . . . depend mainly upon the demand and supply of labour; or . . . on the proportion between population and capital. By population is here meant the number only of the labouring class, or rather of those who work for hire; and by capital only circulating capital, and not even the whole of that, but the part which is expended in the direct purchase of labour. To this, however, must be added all funds which, without forming a part of capital, are paid in exchange for labour, such as the wages of soldiers, domestic servants, and all other unproductive labourers. There is unfortunately no mode of by one familiar term, the aggregate of what has been called the wages-fund of a country: and as the wages of productive labour from nearly the whole of that fund, it is usual to overlook the smaller and less important part, and to say that wages depend on population and capital . . . .
>
> Wages (meaning, of course, the general rate) cannot rise, but by an increase of the aggregate funds employed in hiring, or a diminution in the number of the competitors for hire; nor fall, except either by a diminution of the funds devoted to paying labour, or by an increase in the number of labourers to be paid." (Mill [15] pp.343-344, Bk.II, Ch.XI, § 2.)

Clearly, Mill does not accept the classical school's classification of labor, productive and unproductive to this extent. Therefore, we may interpret demand for labor and supply of labor as aggregates. In addition, Mill regards "wages" as general wage "rates," while he does not specify wage rates, in turn, whether in terms of real or nominal.

Here, let us see the dimension of nominal wage rate and real wage rate. Assuming wages are paid during a month (t), we may formulate nominal wage rate generally as follows.

$$\mathbf{dim}\ (\omega) = \left[nom,\ T \in t\left(\tau_0 \rightarrow \tau_1\right)\right] \qquad (3.1.1)$$

While dimension of the price of wage goods is

$$\dim\left(p\right)=\left[\frac{nom}{real},\ T\in\tau_0\right] \tag{3.1.2}$$

Thus, generally speaking, the real wage rate has the dimension of the real and a flow.

$$\dim\left(\frac{\omega}{p}\right)=\left[real,\ T\in t\left(\tau_0\rightarrow\tau_1\right)\right] \tag{3.1.3}$$

Every economist would accept the formulations of real and nominal rates of wage above as common sense. If Mill would consider his wages-fund theory as determining apparatus of nominal rate of wages, the dimension of it should be

$$\dim\left(\omega\right)=\frac{\left[real\left(K\right),\ T\in\tau_0\right]}{\left[real\left(L\right),\ T\in\tau_0\right]}=\left[real\left(K/L\right),\ T\in\tau_0\right] \tag{3.1.4}$$

Here, K stands for a unit of the wages-fund, which is a stock from definition, L for the number of people hired. That is to say, as long as the wages-fund is a stock, there is no mechanism to determine the general rate of wage, nominal or real, for there is no time interval in (3.1.4) and capital-labor ratio in itself is not of nominal dimension.

Instead, when wages-fund is a flow (in Mill's own word, *circulating capital*),

$$\dim\left(\omega\right)=\frac{\left[real\left(K\right),\ T\in t\left(\tau_0\rightarrow\tau_x\right)\right]}{\left[real\left(L\right),\ T\in\tau_0\right]} \tag{3.1.5}$$

Even by this mode, the inconsistency of dimension remains, for (3.1.1) and (3.1.5) are not the same. To avoid it, we regard wages-fund K as an amalgamation of past labor, $L(l,\delta)$, l denotes the own-rate of interest of labor and δ the rate of depreciation of labor (past dead labor) in terms of labor.

$$\dim\left(\omega\right)=\frac{\left[real\left(L\left(l,\delta\right)\right),\ T\in t\left(\tau_0\rightarrow\tau_x\right)\right]}{\left[real\left(L\right),\ T\in\tau_0\right]} \tag{3.1.6}$$

Clearly, we do have no equations to determine l and δ uniquely in this system. Moreover, more importantly, (3.1.5) tells us nothing about time duration starts at τ_0 and ends at τ_x. This holds true the case of real wage rate. In a word, what is called classical wages-fund doctrine is a wrong theory in confusing stock and flow. That is to say, even if it is possible to determine the amount of capital K, the general wage rate determined by the proportion of capital and labor is ambiguous, namely, whether the rate is expressed in terms of hour, day, month, etc., cannot specified.

In fact, this point was made clear by Charles Morrison (Morrison [16]) and revaluated by Fisher (Fisher [8]). Another issue suggested by Morrison and Fisher implies possible interpretation of the wages-fund controversy; the role and significance of stock-flow relationship. Negishi gets the root of the non-Walrasian tradition in the equilibrium theories from Thornton's criticism to the wages-fund doctrine. We may find another implication from Morrison's criticism, that is, the Newcomb-Fisher tradition. Even if the reason of recantation of the wages-fund theory is unique, the meaning of it is not unique.

3.2. Utility Function and Income Restraint

Almost every student in economics learns the theory of consumer's behavior in which the optimization procedure plays an important role. In this section, we see the basic concepts of utility and income; hence, we cannot argue the optimization method in itself in detail.

First, we begin with verifying the dimension of "utility." To assume familiar utility function, we get the next formula.

$$\mathbf{dim}\left(u\right) = \mathbf{dim}\left(x_1^{\alpha} \cdot x_2^{\beta}\right) = \left[real\left(A^{\alpha} \cdot B^{\beta}\right),\ T \in \tau_1\right] \tag{3.2.1}$$

Here, A and B stand for any measuring units of two goods; notice that we define utility not as a psychological quantity satisfied through time interval but as our estimation for goods in our hands now, and that the unit of measurement of utility depends on α and β, so that the unit is not of a unique dimension.

Moreover, we adopt the normal assumption of α and β, namely, $\alpha+\beta=1, \alpha>0$, and $\beta>0$. Nevertheless, note that the measuring unit of utility solely depends on α. Thus, as long as we accept assumptions above, the choice of unit of utility may be arbitrary or *ad hoc.*

Next, we deal with income restraint.

$$\mathbf{dim}\left(y\right) = \left[nom,\ T \in t_0\left(\tau_0 \rightarrow \tau_1\right)\right] \tag{3.2.2}$$

$$\begin{aligned}\mathbf{dim}\left(p_1 x_1 + p_2 x_2\right) &= \left[\frac{nom}{real\left(A\right)} real\left(A\right), T \in \tau_1\right] + \left[\frac{nom}{real\left(B\right)} real\left(B\right),\ T \in \tau_1\right] \\ &= \left[nom,\ T \in \tau_1\right]\end{aligned} \tag{3.2.3}$$

Evidently, we cannot equate (3.2.2) a flow quantity (pecuniary income) with (3.2.3) a stock quantity (amount of expenditure). Hence, we redefine income as a stock, say, cash balance on Monday[13] without considering on their source and/or the rate of interest.

$$\dim(y) = [nom,\ T \in \tau_1] \tag{3.2.4}$$

By means of such a slightly odd manipulation, we can determine the shape of demand curves. To solve the optimization problem, we get

$$x_1 = \frac{\alpha y}{p_1},\ \ x_2 = \frac{\beta y}{p_2} \tag{3.2.5}$$

It is evident from (3.2.5) that α and β are of the dimension of pure number. Indeed, they are ratio of expenditure to income; each of them is a stock quantity in terms of nominal unit. If this argument is correct, then we must comprehend demand for any commodity is a stock quantity.

Thus, the familiar theory of rational consumer's behavior stands on the some uncertain basis; the ambiguous measuring unit of utility[14] and the income converted arbitrarily a stock quantity on Monday.

3.3. Production Functions and Distribution

In this section, we deal with what is called production functions and its corollary, the theorem of perfect functional distribution or exhaustion of the total product.

Famous Cobb-Douglas production function is originally formulated as such.

$$P = bL^k C^{1-k} \tag{3.3.1}$$

Paul Douglas himself, according to his memoirs (Douglas [6]), devoted to estimate k rather than L (labor) and C (capital), for Charles W. Cobb, Douglas' friend and a mathematician, suggested him to use Euler's theorem of a homogeneous function of the first degree[15] for his purpose.

Using our common notation, let us see dimensions of each variable and parameter.

$$Y = L^\alpha K^\beta \quad \alpha + \beta = 1, \alpha > 0, \beta > 0 \tag{3.3.2}$$

$$\begin{aligned}
\dim(Y) &= [real\,(A),\ T \in t_0(\tau_0 \rightarrow \tau_1)] \\
\dim(L) &= [real\,(L),\ T \in t_0(\tau_0 \rightarrow \tau_1)] \\
\dim(K) &= [real\,(X),\ T \in \tau_0] \\
\dim(\alpha) &= [pure] \\
\dim(\beta) &= [pure]
\end{aligned} \tag{3.3.3}$$

Unquestionably, no matter what means the dimension of amount of production (Y) harmonize with those of dimensions of the other variables and parameters. Especially, note that capital stock invested at the starting point of the production period does not have a common unit of measurement, even if, as Joan Robinson ironically named, “leets” are introduced.

Moreover and more importantly, parameters α and β are of the dimension of pure number ***de definitio***. Remember, however, according to the Euler’s theorem, they are also dividends to labor and capital ***de facto***, so that

$$\alpha = \frac{\partial Y}{\partial L} = \alpha \left(\frac{K}{L} \right)^{\beta} \tag{3.3.4}$$

and

$$\beta = \frac{\partial Y}{\partial K} = \beta \left(\frac{L}{K} \right)^{\alpha} \tag{3.3.5}$$

Aside from a tautological nature of this system of equations, the next constraint must hold true to keep logical consistency.

$$\alpha \left(\frac{K}{L} \right)^{\beta} + \beta \left(\frac{L}{K} \right)^{\alpha} = 1 \tag{3.3.6}$$

Again, unity on the right side is clearly of the dimension of pure number, so that the terms on left side must be of the same dimension. Or, if capital-labor ***ratio*** and labor-capital ***ratio*** weights α and β respectively, the sum must remain *pure number* 1. That is,

$$\mathbf{dim} \left[\left(\frac{K}{L} \right)^{\beta} \right] = [pure]$$

$$\mathbf{dim} \left[\left(\frac{L}{K} \right)^{\alpha} \right] = [pure] \tag{3.3.7}$$

Being K and L of the different dimensions as already noted, the condition (3.3.7) is never satisfied. Even if we regard K as an amalgamation of the past labor, as we did in discussed on the wages-fund doctrine above, the discrepancy of the dimensions in denominator and numerator remains. To sum up, the Cobb-Douglas production function is, even though mathematically useful and consistent, self-contradictory as economic theory radically.

Moreover, as Douglas regretted in his revised preface in 1957 (Douglas [6]), if we introduce “technology” and “the intangible, but real influence of morale” into the production function, it will lead to further confusion as to time and dimension.

Similar argument holds true in the case of the other types of production functions.

First, the CES type

$$Y = [\alpha L^{-\rho} + \beta K^{-\rho}]^{\frac{1}{\rho}} \tag{3.3.8}$$

of which a special case (ρ=0) is the Cobb-Douglas type mentioned above.

Second, a special case ($\rho=\infty$) of the CES type is called the Leontief type.

$$Y = \min\left[\alpha L,\ \beta K\right] \tag{3.3.9}$$

Finally, a special case ($\rho=-1$) of the CES type is the linear type.

$$Y = \alpha L + \beta K \tag{3.3.10}$$

It is easy to show that the quite same inconsistency will emerge in each type of production function.

3.4. The Conception of IS-LM Analysis

Since J.R. Hicks [11] "suggested" IS-LL curves as an interpretation of the novelty of Keynes' *General Theory,* the tool has been accepted widely. Although not a few theorists their doubts on the theoretical background of IS-LM[16], as far as the author see, no one can find its defect in the light of dimensions and time.

Hicks presents his view on the difference of Keynes and the classical school by means of the system of equations (Hicks [11] pp.151-153).

The system of the classical school

$$M = kY,\ I = I(i),\ I = S = S(i,\ Y) \tag{3.4.1}$$

The system of Keynes

$$M = L(i),\ I = I(i),\ I = S = S(Y) \tag{3.4.2}$$

Here, M stands for money supply, k for Marshallian k, Y for national income, I for investment, S for savings, i for the rate of interest, and L for liquidity preference or demand for money[17].

(3.4.1) and (3.4.2) show sharp contrast in two points; first, on supply for and demand of money, the system of the classical school accepts what is called the Cambridge cash balance equation while Keynes denies it and adopts the theory of Liquidity preference, second, the classics uses the loanable-funds theory in determination of I and S, while Keynes stresses upon the multipliers process.

Hicks takes a further step from (3.4.1) and (3.4.2) to the *general* theory, which contains next three equations.

$$M = L(Y,\ i),\ I = I(i),\ I = S = S(Y) \tag{3.4.3}$$

LM (in Hicks' own words *LL*) curve slopes upwards.

$$dM = \frac{\partial L}{\partial Y} dY + \frac{\partial L}{\partial i} di = 0 \tag{3.4.4}$$

where noticing that

$$\frac{\partial L}{\partial Y} > 0 \;\; and \;\; \frac{\partial L}{\partial i} < 0 \tag{3.4.5}$$

therefore,

$$\frac{di}{dY} = -\left\{ \frac{\partial L}{\partial Y} \middle/ \frac{\partial L}{\partial i} \right\} > 0 \tag{3.4.6}$$

The slop of *IS* curve is downwards.

$$\frac{\partial I}{\partial i} di = \frac{\partial S}{\partial Y} dY \tag{3.4.7}$$

hence,

$$\frac{di}{dY} = \left\{ \frac{\partial S}{\partial Y} \middle/ \frac{\partial I}{\partial i} \right\} < 0 \tag{3.4.8}$$

Thus the equilibrium point (Y^*, i^*) is determined uniquely and *simultaneously* by the crossing curves, LM curve and IS curve. According to Hicks' own words, "Income and the rate of interest are *now determined* together at ··· the point of intersection of the curves LL and IS." (Hicks [11] p.153, italics added.)

Here, to turn our attention to the time dimension and the units of measurement, we get,

$$\begin{aligned} \dim(M) &= [nom,\, T \in \tau_0] \\ \dim(L) &= [nom,\, T \in \tau_0] \end{aligned} \tag{3.4.9}$$

We may say that, as to the LM curve, the equilibrium locus is of nominal and stock dimensions.

On IS curve, Hicks does not explicitly refer to the unit of account of ***Y***, ***I***, and ***S***. To consider, however, his formulization of the Cambridge cash balance equation in (3.4.2), we may regard the unit of ***Y*** as a nominal number, for Hicks omitted the general price level (***P***) of nominal income. To keep the system consistent, both ***S*** and ***I*** should be also of nominal dimension.

$$
\begin{aligned}
\dim(Y) &= \left[nom,\ T \in t_0(\tau_0 \rightarrow \tau_1)\right] \\
\dim(I) &= \left[nom,\ T \in t_0(\tau_0 \rightarrow \tau_1)\right] \\
\dim(S) &= \left[nom,\ T \in t_0(\tau_0 \rightarrow \tau_1)\right] \\
\dim(i) &= \left[pure,\ T \in \frac{1}{t(\tau_0 \rightarrow \tau_1)}\right]
\end{aligned}
\tag{3.4.10}
$$

It is evident ***I*=*S*** locus shows a flow equilibrium. Therefore, the simultaneous determination of national income and the rate of interest by means of the IS-LM apparatus implies the *simultaneous* treatment of stocks-equating condition and flows-equating condition. We cannot equate stock variables, which exist at the point of time, with flows, which exist during the interval of time, as if they exist at the exactly same point of time. As mentioned above, we may, of course, take flows back to the present beings ***via*** capitalization. This procedure, however, needs to assume the rate of interest ***a priori***, which is determined at the intersection point of IS-LM *a posteriori.*

Moreover, the equilibrium point (Y^*, i^*) contains time duration clearly. This holds true on the IS curve, while the LM curve is of the stock dimension. To fix the equilibrium point (Y^*, i^*) ***simultaneously***, we should keep ***L*=*M*** condition unchanged from the point of time τ_0 on until equilibrium point (Y^*, i^*) is determined. Even if we wait the time interval of the adjusting process in ***I*** and ***S***, their tâtonnement process may disturb the condition ***L*=*M*** through the changes of the rate of interest and/or national income, furthermore, through the credit creation that finances the changes of ***I*** and/or ***S***.

4. Concluding Remarks

Thus far, we define stocks, flows and the unit of measurement, and attempt to apply them to some of well-known economic theory. Mathematical model buildings often used are of significance as long as they aim to attain reasonable inference in the restricted boundary. As noted above, the boundary is to some extent narrow for economists. When they equate an economic quantity with another economic quantity, they should notice not only its unit of measurement but also its time dimension, stock and flow.

Indeed, indifference to them causes some critical problems. In the classical wages-fund doctrine, there is no mechanism to determine the *rate* of wage, ***i.e.***, period wages paid. The familiar utility theory lacks a unique unit of measurement of utility. The production functions are inconsistent with the assumption of the linear homogeneous (or homogeneous of the first degree) character and the theorem of exhaustion of the total product. Finally, IS-LM analysis is self-contradictory as to *simultaneous* treatment and determination of stocks and flows.

If the argument developed in this article were correct, we might need a Copernican change in our way of thinking on the *modus operandi* of economic quantities. Admittedly, the topics treated here are too few to proof the meaning and significance of the conception of dimension, stocks and flows to full extent. Nevertheless, the problems presented here are open to further arguments and reexaminations. If needed, all the notable axiomatic economic theories and positive economic models may be reconsidered in terms of the conceptions presented here.

(Associate Professor, The Faculty of Economics, Takasaki City University of Economics)

Notes

1 The author wishes to appreciate especially my colleague, Mr. NAKANO Masahiro who gives significant comments on early draft of this paper. It is no need to say that all mistakes and ambiguities remaining in this paper, if any, are mine.

2 Unfortunately, they encompass neither Irving Fisher's economics, such as Fisher [10], nor its predecessor, Simon Newcomb, one of founders of stock-flow conceptions (Newcomb[18] Esp. Book IV). Fisher's economics developed around "time" concept. The author is preparing to publish a paper on Newcomb's Economics.

3 As we shall see later, a reason of Allen and Patikins' error is their neglect that we say *rate* of interest rather than *ratio* of interest.

4 To avoid this difficulty, we need to introduce the price concept, namely the tool convert real unit to money unit, however determined, to aggregate heterogeneous goods.

5 As we will see later, this is the reason why there should be a discount rate and/or the stock-flow tension. In addition, such an intertemporal condition involves risk and uncertainty. Consideration to them is, however, beyond the aim of this paper.

6 Therefore, we could not accept Patinkin's assertion that a stock quantity has no time dimension, and a flow-stock tension is an illusion. (See [19] Ch.II, fn.15 and Appendix XI.) Were his insist correct, Keynes's famous real interest rates or own-rates of interest concept would be meaningless (Keynes [14] Ch.XVII). Though we have no space to discuss on it in detail, it is important to suggest that it is a key conception to appropriate understanding of Fisher-Sraffa-Keynes connection around the conception of the own-rates of interest. Patinkin's misunderstanding seems to stem from R.G.D.Allen's basic fallacy that the rate of interest is a pure number, thus it has no time dimensions. See Allen [2] § 9.7.

7 The idea adopted here owes to Fisher [6] pp.513-516.

8 In this instance, we accept Patinkin's assumption ([19] p.517) that annuity payment or cash flow is not of flow dimension but exists at the each starting point of the intervals in question.

9 The rate of interest of the former case may be called agio or premium sense; that of latter price sense.

10 This idea is originally found in Jevons[13] pp.245-247. He made, however, a mistake that R and S are of the same time dimension, while he defines R is a flow and S is a stock.

11 In U.K., this concept is often used in valuation in land.

12 As Negishi (Negishi [17]) points out, however, this is not part and parcel of Thornton's criticism. Negishi sets the record straight by pointing out that the real issue consists in Mill's labor market analysis in terms of supply of and demand for it.

13 This seems to be a reason why J.R.Hicks defines his temporal equilibrium in terms of "a week." On his definition of week, see Hicks [12] p.122. In addition, Hicksian income concept is called "accrual type", in contrast to "disposition type" largely maintained by Irving Fisher. On this classification of income, see a classic survey article, Wueller [23].

14 Though a different method and scope is adopted, a famous paper is worth for re-examination in this context, see Alchian [1].

15 His estimated value of k is 0.75. See also Cobb & Douglas [4] and Douglas [6].

16 *E.g.,* see Young [21], and Young et al [22].

17 Although we follow the Hicksian way of thinking, the notation is not Hick's own. For convenience and simplicity, we observe our common usage.

【References】

[1] Alchian, Almen A. (1953) "The Meaning of Utility Measurement," *American Economic Review* 58-1.

[2] Allen, Roy George Douglas (1938) *Mathematical Analysis for Economists.* Macmillan.

[3] Brown, Murray (1967) *The Theory and Empirical Analysis of Production.* NBER, Vol. 31 of *Studies in Income and Wealth.* Columbia University Press.

[4] Cobb, Charles and Douglas, Paul Howard (1928) "A Theory of Production," *American Economic Review* Suppl. XVIII.

[5] Currie, Martin and Steedman, Ian (1990) *Wrestling with Time: Problems in Economic Theory.* The University of Michigan Press. Ann Arbor.

[6] Douglas, Paul Howard (1934) *The Theory of Wages.* Macmillan. Reprint in 1957, Kelly and Macmillan.

[7] —— (1967) "Comments on the Cobb-Douglas Production Function," in Brown ed. [3].

[8] Fisher, Irving (1896) "What is Capital ?" *Economic Journal* 6. In W. Barber et al. ed. *Works of Irving Fisher* Vol. 1. Pickering and Chatto, 1997.

[9] —— (1897) "The Rôle of Capital in Economic Theory." *Economic Journal* 7. In W. Barber et al. ed. *Works of Irving Fisher* Vol. 1. Pickering and Chatto, 1997.

[10] —— (1906) *The Nature of Capital and Income.* In W. Barber et al. ed. *Works of Irving Fisher* Vol. 2. Pickering and Chatto, 1997.

[11] Hicks, John Richard (1937) "Mr. Keynes and the 'Classics': A Suggested Interpretation," *Econometrica* 5-2.

[12] ——(1939) *Value and Capital; An Inquiry into Some Fundamental Principles of Economic Theory.* Clarendon Press. Second ed. 1946.

[13] Jevons, William Stanley (1871) *The Theory of Political Economy.* Fifth ed.1957, A.M.Kelly.

[14] Kenyes, John Maynard (1936) *The General Theory of Employment, Interest, and Money.* Macmillan. *Collected Works of John Maynard Keynes,* Vol.VII. Macmillan.

[15] Mill, John Stuart (1848) *Principles of Political Economy with Some of Their Applications to Social Philosophy.* Ed. by W.J.Ahley, Kelly.

[16] Morrison, Charles (1854) *Essay on the Relations between Labour and Capital.* Longman. Reprint Arno Press, 1972.

[17] Negishi Takashi (1986) "Thornton's Criticism of Equilibrium Theory and Mill," *History of Political Economy* 18-4.

[18] Newcomb, Simon (1886) *Principles of Political Economy.* Harper & Bros. Reprint A.M.Kelly.

[19] Patinkin, Don (1965) *Money, Interest, and Prices: An Integration of Monetary and Value Theory.* Harper and Row. Second ed.

[20] Thornton, William Thomas (1869) *On Labour, Its Wrongful Claims and Rightful Dues; Its Actual Present and Possible Future.* Macmillan.

[21] Young, Warren (1987) *Interpreting Mr Keynes; The IS-LM Enigma.* Basil Blackwell.

[22] Young, Warren and Zilberfarb, Ben eds. (2000) *IS-LM and Modern Macroeconomics.* Kluwer Academic Publishers.

[23] Wueller, Paul H. (1938) "Concept of Taxable Income (I-III)" *Political Science Quarterly* 53-54. Partly reprinted in Parker & Harcourt eds. *Readings in the Concept and Measurement of Income.* 1969.