Versioning in Monopoly and Oligopoly

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Abstract

The objective of this paper is to explain strategic product choices by firms in oligopoly when they can produce several versions of a product such as electronics with different sizes in memories. In this paper we assume firms can choose two types of one product: high quality product with high marginal cost and low quality product with low marginal cost. We consider heterogeneous consumers in terms of the premium they are willing to pay for the high quality product. First, firms simultaneously choose from three choices: only low quality product, only high quality product, or both types of the product (versioning). Then firms choose prices simultaneously.

As a benchmark, we first analyze a monopoly. A monopolist never chooses to sell only high quality product. It sells only low quality product if the difference in the marginal costs is greater than the highest value of the premium consumers are willing to pay, and sells both products otherwise.

In duopoly, if the highest value of the premium is less than half of the difference in the marginal costs, any strategy profile except for both firms choosing only high quality product can be an equilibrium. If the highest value of the premium is greater than the half of the difference in the marginal costs, the equilibrium is either (1) one firm chooses only low quality product and the other chooses high quality product (market segregation) or (2) both firms choose both products.

In three-firm oligopoly, if the highest value of the premium is less than the difference in the marginal costs, any strategy profile except for all firms choosing only high quality product can be an equilibrium. Otherwise, any strategy profile except for all firms choosing only high quality product and all firms choosing only low quality product can be an equilibrium.

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1 Introduction

Versioning, also called quality discriminations or menu pricing, is the way for a producer to provide different qualities of a good at different prices. Although this term is mainly used in informational goods, such as softwares and applications, it can be applied to any products such as electronics (256GB versus 64GB), books (hard cover versus paperback), even agricultural products (organic versus conventional). It can also be applied to a product with different impacts on the environment or the society. In this paper we simply refer them as high quality product and low quality product.

The main purpose of this paper is to analyze versioning as a firm's strategy to differentiate itself from others. In US baby food industry, for example, Beech Nut offers only conventional baby food, Earth's Best offers only organic baby food, and Gerber offers both organic and conventional baby food. Our main goal is to explain how this market was formed as an equilibrium.

The choice can be influenced by the costs of production and consumer tastes. Usually high quality products cost more to produce and the consumers are willing to pay extra for the high quality. Even if the product itself is identical, if products are made with different levels of the impact on the environment, some consumers are willing to pay more for products made with an environmentally friendly method. Some consumers are willing to pay more for goods and services from companies that have implemented programs to give back to society.

Research on versioning has not yet widely been done. Varian (1997) and Belleflamme (2005) provided a framework to analyze versioning in a monopoly for information goods. Belleflamme (2005) showed that when the consumers' utility for an information good can be separated along two dimensions, a "key dimension" for which consumers have different valuations, and a "secondary dimension" for which all consumers have the same valuation, versioning the good along the key dimension is the most profitable option for the monopolist. Diaw and Pouyet (2004) analyze a duopoly in which firms offer differentiated goods to a representative consumer and show that firms prefer not price discriminate when there is an asymmetric information about consumer's taste.

Our setting can also be categorizes as vertical product differentiation. There are numerous papers about vertical product differentiation in Bertrand oligopoly. Shaked and Sutton (1983) analyze oligopoly with vertical differentiation and show that there may exist an upper bound for the number of firms which can coexist in the market. Greenstein and Ramey (1998) analyzes product innovation as a mean of vertical product differentiation and show that firms have strictly greater incentive to innovate when the old product market is a monopoly with a threat of entry than competition. Tanaka (2001) analyzes a duopoly with vertical product differentiation and shows that a quantity strategy gives higher profit than a price strategy, as in Singh and Vives (1984) for horizontal product differentiation. Zanchettin (2006) shows the opposite result when asymmetry in the costs and demands is strong. Cheng, et al. (2011) analyzes a duopoly which choose the number of products and qualities of products in the first stage and prices in the second stage and show that the profits for firms in a multiproduct duopoly are lower than that in a single product duopoly.

However, in many papers the product quality choices are fixed or continuous, and the market structure is either a monopoly or a duopoly. In our paper, we make it simpler: product quality choices are restricted to two, since our focus is on firm's behavior of choosing versioning as a way to differentiate itself from other competitors. We also analyze three firm oligopoly.

Our model follows the basic Hotelling setting: a continuum of consumers with density 1 have different willingness to pay for the premium for high quality products. The willingness to pay for the premium is distributed uniformly over the interval $[0, \bar{\rho}]$. The willingness to pay for low quality product is the same for all consumers. Each consumer purchases one unit of the product or does not purchase at all. Firms choose the product type (high quality, low quality, or both qualities (versioning)) in the first stage and the prices in the second stage. The equilibrium concept we use is subgame-perfect equilibrium. As a benchmark, we first analyze a monopoly. Then we analyze duopoly and three-firm oligopoly.

Our results are the following. In monopoly, a monopolist can offer only one type of the product or two types of the product as a versioning, but it never chooses to sell only high quality product. It sells only low quality product if the difference in the marginal costs is greater than the highest value of the premium consumers are willing to pay, $\bar{\rho}$, and sells both products otherwise.

In duopoly, if the highest value of the premium is less than half of the difference in the marginal costs, any strategy profile except for both firms choosing only high quality product can be an equilibrium. If the highest value of the premium is greater than the half of the difference in the marginal costs, the equilibrium is either (1) one firm chooses only low quality product and the other chooses high quality product (market segregation) and both earn positive profits or (2) both firms choose both products and earn zero profit. The firms earn strictly greater profits in (1) than in (2), and this result is consistent with Cheng, et al. (2011).

In three-firm oligopoly, if the highest value of the premium is less than the difference in the marginal costs, any strategy profile except for all firms choosing only high quality product can be an equilibrium. Otherwise, any strategy profile except for all firms choosing only high quality product and all firms choosing only low quality product can be an equilibrium. This explains the US baby food market and many more. As the number of firms increases from two to three, the profit each firm earns either stays at 0 or decreases.

The rest of the paper is organized as follows. Section 2 outlines our basic model. Section 3 analyzes monopoly, section 4 analyzes duopoly, and section 5 analyzes three-firm oligopoly. Section 6 concludes our results.

2 Basic Model

We consider a market with a vertical product differentiation. For simplicity, we restrict the product choices to two product types: high quality product H or low quality product L. These can also be interpreted as environmentally friendly and conventional, deluxe and original, or branded product and generics. The marginal cost of producing H is denoted as c_H and the marginal cost of producing L is denoted as c_L. We assume $c_L < c_H$.

We assume there are a continuum of consumers with density 1. All consumers are willing to pay $\theta > 0$ for low quality product. We assume $\theta > c_H$. Consumers differ in terms of their willingness to pay for the premium for high quality product. Consumer i's willingness to pay for the premium is denoted as ρ_i, and the willingness to pay for the premium is distributed uniformly over the interval $[0, \bar{\rho}]$. Thus consumer i's willingness to pay for high quality product is written as $\theta + \rho_i$. Each consumer buys one unit of a product or buys none.

In this paper, we consider monopoly, duopoly, and three-firm oligopoly. Each firm chooses the menu of products simultaneously: high quality product only, low quality product only, or both (versioning). Then each chooses its price (or prices) simultaneously as in a Bertrand oligopoly.

3 Monopoly

At first, we analyze a monopoly as a benchmark. There are 3 possible options for the monopolist: Option (1) Offer only low quality product, Option (2) Offer only high quality product, and Option (3) Offer both products.

Option (1) Offer only low quality product

The highest possible price monopolist can charge is the consumers' reservation price θ. The profit is $\pi_L^m = p_L^m - c_L = \theta - c_L$.

Option (2) Offer only high quality product

Suppose the monopolist charges p_H for high quality product. Then there is a consumer who is indifferent between buying high quality product and buying nothing, so let us call him $\hat{\rho}$. For $\hat{\rho}$, the net utility from buying the high quality product is $\theta + \hat{\rho} - p_H$ and the net utility from buying nothing is 0. Therefore, $\hat{\rho} = p_H - \theta$. Note that if $p_H = \theta$, then $\hat{\rho} = 0$ and thus all the consumers buy the high quality product, so the profit is $\pi_H^m = \theta - c_H$. Charging $p_H < \theta$ makes $\hat{\rho} < 0$ but the demand stays the same so there is no incentive to charge $p_H < \theta$. Suppose the monopolist charges $p_H > \theta$. Since only consumers who have higher valuations than $\hat{\rho}$ will buy the product, the demand is $D_H(p_H) = \frac{\bar{\rho} - (p_H - \theta)}{\bar{\rho}}$. The monopolist chooses p_H to maximize its profit $\pi_H = (p_H - c_H)D_H(p_H)$. The profit maximizing price is $p_H^m = \frac{\theta + \bar{\rho} + c_H}{2} > c_H$ and the maximum profit is $\pi_H^m = \frac{1}{4\bar{\rho}}(\theta + \bar{\rho} - c_H)^2$. Note that $p_H^m \geq \theta$ if $\bar{\rho} \geq \theta - c_H$. Therefore, we can conclude that the monopolist charges $p_H^m = \theta$ if $\bar{\rho} < \theta - c_H$ and $p_H^m = \frac{\theta + \bar{\rho} + c_H}{2}$ if $\bar{\rho} \geq \theta - c_H$.

Option (3) Offer both products

Suppose the monopolist charges p_L for low quality product and p_H for high quality product. If $p_L > \theta$, no consumers buy low quality product, so it is identical to Case (2). Suppose the monopolist charges $p_L \leq \theta$. There is a consumer who is indifferent between buying low quality product and high quality product, and let us call him $\tilde{\rho}$. For $\tilde{\rho}$, the net utility from buying high quality product is $\theta + \tilde{\rho} - p_H$ and the net utility from buying low quality product is $\theta - p_L$. Therefore, $\tilde{\rho} = p_H - p_L$.

The demand for low quality product is $D_L(p_L, p_H) = \frac{p_H - p_L}{\bar{\rho}}$ and the demand for high quality product is $D_H(p_L, p_H) = \frac{\bar{\rho} - (p_H - p_L)}{\bar{\rho}}$. The monopolist chooses p_L and p_H to maximize its profit $\pi_V = (p_L - c_L)D_L(p_L, p_H) + (p_H - c_H)D_H(p_L, p_H)$. Since $\frac{\partial \pi_V^m}{\partial p_L} = 0$ and $\frac{\partial \pi_V^m}{\partial p_H} = 0$ are satisfied simultaneously only if $\bar{\rho} = 0$, we have to consider several cases.

If $\frac{\partial \pi_V^m}{\partial p_L} = 0$ and $\bar{\rho} > 0$, $\frac{\partial \pi_V^m}{\partial p_H} > 0$. This implies that the profit is increasing in the price of high quality product and thus profit maximizing price for the high quality product is $p_H^m = \theta + \bar{\rho}$. This means no one (except for the consumer with $\bar{\rho}$) buys the high quality product and thus the profit maximizing price for the low quality product is $p_L^m = \theta$ and the maximum profit is $\pi_V^m = \theta - c_L$.

If $\frac{\partial \pi_V^m}{\partial p_H} = 0$ and $\bar{\rho} > 0$, $\frac{\partial \pi_V^m}{\partial p_L} > 0$. Therefore the profit maximizing price for low quality product is $p_L^m = \theta$. Then the profit maximizing price for high quality product is $p_H^m = \frac{1}{2}(\bar{\rho} + c_H - c_L) + \theta$. If $\bar{\rho} < c_H - c_L$, $p_H^m > \theta + \bar{\rho}$, thus no one buys high quality product. The profit is $\pi_V^m = \theta - c_L$. If $\bar{\rho} \geq c_H - c_L$, $p_H^m \leq \theta + \bar{\rho}$, thus the consumers with $\rho \geq \tilde{\rho}$ will buy high quality product. The profit is $\pi_V^m = \frac{1}{4\bar{\rho}}(\bar{\rho} - (c_H - c_L))^2 + \theta - c_L$.

Now consider which product (products) the monopolist should offer. If $\bar{\rho} < \theta - c_H$ and $\bar{\rho} < c_H - c_L$, $\pi_L^m = \theta - c_L$ for Option (1), $\pi_H^m = \theta - c_H$ for Option (2), and $\pi_V^m = \theta - c_L$ for Option (3). By choosing Option (1) or (3), the monopolist only sells low quality product.

If $\bar{\rho} < \theta - c_H$ and $\bar{\rho} \geq c_H - c_L$, $\pi_L^m = \theta - c_L$ for Option (1), $\pi_H^m = \theta - c_H$ for Option (2), and $\pi_V^m = \frac{1}{4\bar{\rho}}(\bar{\rho} - (c_H - c_L))^2 + \theta - c_L$ for Option (3). Obviously, Option (3) gives the monopolist the highest profit by selling both products.

If $\bar{\rho} \geq \theta - c_H$ and $\bar{\rho} < c_H - c_L$, $\pi_L^m = \theta - c_L$ for Option (1), $\pi_H^m = \frac{1}{4\bar{\rho}}(\theta + \bar{\rho} - c_H)^2$ for Option (2), and $\pi_V^m = \theta - c_L$ for Option (3). It is easy to show that $\pi_V^m > \pi_H^m$ under those conditions. The monopolist sells only low quality product.

If $\bar{\rho} \geq \theta - c_H$ and $\bar{\rho} \geq c_H - c_L$, $\pi_L^m = \theta - c_L$ for Option (1), $\pi_H^m = \frac{1}{4\bar{\rho}}(\theta + \bar{\rho} - c_H)^2$ for Option (2), and $\pi_V^m = \frac{1}{4\bar{\rho}}(\bar{\rho} - (c_H - c_L))^2 + \theta - c_L$ for Option (3). Again, it is easy to show that $\pi_V^m > \pi_H^m$ and $\pi_V^m > \pi_L^m$ under those conditions. Therefore, the monopolist sells both products.

Combining all the results tells us that the monopolist will never choose Option (2) and the monopolist offers only the low quality product if $\bar{\rho} < c_H - c_L$ and both products if $\bar{\rho} \geq c_H - c_L$. Our result is opposite to Belleflamme (2005), who restricted the focus only on information goods with considerably low marginal costs and showed that under their conditions the monopolist chooses high quality product if he decides to sell a single quality. Belleflamme (2005) also showed that when the consumers' utility for an information good can be separated along two dimensions, versioning the good along the key dimension is the most profitable option for the monopolist. We do not require consumers' utility to have two dimensions and successfully showed that versioning is profitable for a monopolist under much simpler setting.

4 Duopoly

We now consider a duopoly. Name the two firms A and B. There are 6 possible cases for duopoly. Case (1) Both firms offer only low quality product, Case (2) Both firms offer only high quality product, Case (3) One firm offers only low quality product and the other offers only high quality product, Case (4) One firm offers only low quality product and the other offers both products, Case (5) One firm offers only high quality product and the other offers both products, and Case (6) Both offer both products. We solve the game with backward induction.

4.1 Bertrand Duopoly

Case (1) Both firms offer only low quality product

In this case, there is no product differentiation and thus both firms charge the marginal cost c_L. The profit for each firm is zero.

Case (2) Both firms offer only high quality product

Similarly to Case (1), both firms charge the marginal cost c_H and earns zero profit.

Case (3) One firm offers only low quality product and the other firm offers only high quality product

Suppose firm A offers low quality product and firm B offers high quality product. Suppose firm A is charging p_L and firm B is charging p_H. The consumer who is indifferent between two firms is $\tilde{\rho} = p_H - p_L$ as in versioning for monopoly. The demand for the low quality product is $D_L(p_L, p_H) = \frac{p_H - p_L}{\bar{\rho}}$ and the demand for the high quality product is $D_H(p_L, p_H) = \frac{\bar{\rho} - (p_H - p_L)}{\bar{\rho}}$. Firm A's profit is $\pi_L^A = (p_L - c_L) D_L(p_L, p_H)$ and firm B's profit is $\pi_H^B = (p_H - c_H) D_H(p_L, p_H)$. Each firm's first order condition gives the best response function: $BR_L(p_H) = \frac{p_H + c_L}{2}$ and $BR_H(p_L) = \frac{p_L + c_H + \bar{\rho}}{2}$. Solving them together gives us the Nash equilibrium $(p_L^*, p_H^*) = (\frac{\bar{\rho} + 2c_L + c_H}{3}, \frac{2\bar{\rho} + c_L + 2c_H}{3})$. The profits are $\pi_L^A = \frac{1}{9\bar{\rho}}(\bar{\rho} + c_H - c_L)^2$ and $\pi_H^B = \frac{1}{9\bar{\rho}}(2\bar{\rho} + c_H - c_L)^2$. The consumer $\tilde{\rho}$ is located at $\frac{\bar{\rho} + c_H - c_L}{3}$.

Note that we have restrictions on the prices: $c_L \leq p_L \leq \theta$ and $c_H \leq p_H \leq \theta + \bar{\rho}$. If $\bar{\rho} < \frac{c_H - c_L}{2}$, $p_H^* = \frac{2\bar{\rho} + c_L + 2c_H}{3} < c_H$, which means firm B charges the price below its marginal cost. Thus the equilibrium prices will instead be $(p_L^*, p_H^*) = (c_H - \bar{\rho}, c_H)$ and the profits are $\pi_L^A = c_H - c_L - \bar{\rho}$ and $\pi_H^B = 0$.[1]

If $\bar{\rho} > 3\theta - c_H - 2c_L$, $p_L^* = \frac{\bar{\rho} + 2c_L + c_H}{3} > \theta$, which makes the demand for low quality product zero. Therefore the equilibrium price will instead be $(p_L^*, p_H^*) = (\theta, \frac{\theta + \bar{\rho} + c_H}{2})$ and the profits are $\pi_L^A = \frac{1}{2\bar{\rho}}(\theta - c_L)(c_H + \bar{\rho} - \theta)$ and $\pi_H^B = \frac{1}{4\bar{\rho}}(\theta + \bar{\rho} - c_H)^2$.

Except for the case with $\bar{\rho} < \frac{c_H - c_L}{2}$ where firm B earns zero profit, both firms earn positive profit in this duopoly due to the product differentiation.

Case (4) One firm offers only low quality product and the other firm offers both products

Suppose firm A offers low quality product and firm B offers both. Both firms charge the marginal cost c_L for low quality product. Firm B charges $p_H^* = BR_H(c_L) = \frac{c_L + c_H + \bar{\rho}}{2}$ if $\bar{\rho} \geq c_H - c_L$, and $p_H^* = c_H$ if $\bar{\rho} < c_H - c_L$ since $BR_H(c_L) < c_H$ for the latter. Firm B's profit for the first case is $\pi_V^B = \frac{1}{4\bar{\rho}}(\bar{\rho} - (c_H - c_L))^2$ and for the second case is 0.

Case (5) One firm offers only high quality product and the other firm offers both products

Suppose firm A offers high quality product and firm B offers both. Both firms charge the marginal cost c_H for high quality product. Firm B charges $p_L^* = BR_L(c_H) = \frac{c_L + c_H}{2} > c_L$ and earns $\pi_V^B = \frac{1}{4\bar{\rho}}(c_H - c_L)^2$ if $\bar{\rho} \geq \frac{c_H - c_L}{2}$, and charges $p_L^* = c_H - \bar{\rho}$ and earns $\pi_V^B = c_H - c_L - \bar{\rho}$ if $\bar{\rho} < \frac{c_H - c_L}{2}$, as we have seen in Case (3).

Case (6) Both firms offer both products

In this case, both firms charge marginal costs for both products and earn zero profit.

4.2 Product Choice

Given the results from the second stage of the game, we can now consider the product choice in the first stage. Each firm chooses one from three actions: Only low quality product, Only high quality product, and Both products. The payoff matrices are shown in Figure 1, 2, 3, and 4. Each firm's best responses are indicated by underline.

Table 1 shows that any product choice except for both firms choosing high quality product can be a pure strategy equilibrium if $\bar{\rho} < \frac{c_H - c_L}{2}$. In other words, if consumers are not willing to pay big premium for high quality product, almost any combination of product choices can

[1] $BR_L(c_H) = \frac{c_H + c_L}{2}$ is not the profit maximizing price for A since this value makes the threshold consumer $\tilde{\rho}$ located beyond $\bar{\rho}$ but the demand stays the same. Firm A just needs to make sure the consumer at $\bar{\rho}$ buys its product.

Firm B

Firm A		Low quality only	High quality only	Both products
	Low	$\underline{0},\ \underline{0}$	$\underline{c_H - c_L - \bar{\rho}},\ \underline{0}$	$\underline{0},\ \underline{0}$
	High	$\underline{0},\ \underline{c_H - c_L - \bar{\rho}}$	$0,\ 0$	$\underline{0},\ \underline{c_H - c_L - \bar{\rho}}$
	Both	$\underline{0},\ \underline{0}$	$\underline{c_H - c_L - \bar{\rho}},\ \underline{0}$	$\underline{0},\ \underline{0}$

Table 1: Duopoly Game when $\bar{\rho} < \frac{c_H - c_L}{2}$

Firm B

Firm A		Low quality only	High quality only	Both products
	Low	$0,\ 0$	$\underline{\frac{(\bar{\rho}+c_H-c_L)^2}{9\bar{\rho}}},\ \underline{\frac{(2\bar{\rho}+c_H-c_L)^2}{9\bar{\rho}}}$	$\underline{0},\ 0$
	High	$\underline{\frac{(2\bar{\rho}+c_H-c_L)^2}{9\bar{\rho}}},\ \underline{\frac{(\bar{\rho}+c_H-c_L)^2}{9\bar{\rho}}}$	$0,\ 0$	$\underline{0},\ \frac{(c_H-c_L)^2}{4\bar{\rho}}$
	Both	$0,\ \underline{0}$	$\frac{(c_H-c_L)^2}{4\bar{\rho}},\ \underline{0}$	$\underline{0},\ \underline{0}$

Table 2: Duopoly Game when $\frac{c_H - c_L}{2} \leq \bar{\rho} < c_H - c_L$

Firm B

Firm A		Low quality only	High quality only	Both products
	Low	$0,\ 0$	$\underline{\frac{(\bar{\rho}+c_H-c_L)^2}{9\bar{\rho}}},\ \underline{\frac{(2\bar{\rho}+c_H-c_L)^2}{9\bar{\rho}}}$	$\underline{0},\ \frac{(\bar{\rho}-c_H+c_L)^2}{4\bar{\rho}}$
	High	$\underline{\frac{(2\bar{\rho}+c_H-c_L)^2}{9\bar{\rho}}},\ \underline{\frac{(\bar{\rho}+c_H-c_L)^2}{9\bar{\rho}}}$	$0,\ 0$	$\underline{0},\ \frac{(c_H-c_L)^2}{4\bar{\rho}}$
	Both	$\frac{(\bar{\rho}-c_H+c_L)^2}{4\bar{\rho}},\ \underline{0}$	$\frac{(c_H-c_L)^2}{4\bar{\rho}},\ \underline{0}$	$\underline{0},\ \underline{0}$

Table 3: Duopoly Game when $c_H - c_L \leq \bar{\rho} < 3\theta - c_H - 2c_L$

		Firm B		
		Low quality only	High quality only	Both products
	Low	0, 0	$\underline{\frac{(\theta-c_L)(c_H+\bar{\rho}-\theta)}{2\bar{\rho}}}$, $\underline{\frac{(\theta+\bar{\rho}-c_H)^2}{4\bar{\rho}}}$	$\underline{0}$, $\underline{\frac{(\bar{\rho}-c_H+c_L)^2}{4\bar{\rho}}}$
Firm A	High	$\underline{\frac{(\theta+\bar{\rho}-c_H)^2}{4\bar{\rho}}}$, $\underline{\frac{(\theta-c_L)(c_H+\bar{\rho}-\theta)}{2\bar{\rho}}}$	0, 0	$\underline{0}$, $\frac{(c_H-c_L)^2}{4\bar{\rho}}$
	Both	$\underline{\frac{(\bar{\rho}-c_H+c_L)^2}{4\bar{\rho}}}$, $\underline{0}$	$\frac{(c_H-c_L)^2}{4\bar{\rho}}$, $\underline{0}$	$\underline{0}$, $\underline{0}$

Table 4: Duopoly Game when $3\theta - c_H - 2c_L \leq \bar{\rho}$

be realized as an equilibrium. In this case, only the firm which is selling low quality product while the other firm is selling only high quality product can earn positive profit.

Table 2, 3, and 4 show that if $\bar{\rho} \geq \frac{c_H-c_L}{2}$, there are only two types of pure strategy equilibria: one firm chooses low quality product and the other chooses high quality product (market segregation) and both firms choose both products (Bertand competition in two markets). Under the first type of equilibrium, both firms earn strictly positive profits. This explains some markets in which high-end firm and low-end firm are coexisting. When both firms choose to offer both products, they earn zero profit. This explains some markets in which all firms are selling several versions of their products, such as electronics including smart phones and tablets. The difference in profits is consistent with Cheng, et al. (2011), although firms choose versioning and offer the interval of qualities which are not overlapping with each other (market segmentation) under specific conditions in their paper. Our results are much simpler and stronger in terms of the applicability to many markets.

5 Three-firm Oligopoly

We now consider an oligopoly with three firms. Name the three firms A, B, and C. There are 10 possible cases for three-firm oligopoly. Case (1) All firms offer only low quality product, Case (2) All firms offer only high quality product, Case (3) One firm offers only low quality product and two firms offer only high quality product, Case (4) One firm offers only high quality product and two firms offer only low quality product, Case (5) One firm offers only low quality product, one firm offers only high quality product, and the last firm offers both products, Case (6) One firm offers only low quality product and two firms offer both products, Case (7) One firm offers only high quality product and two firms offer both products, Case (8) Two firms offer only low quality product and one firm offers both products, Case (9) Two firms offer only high quality product and one firm offers both products, and Case (10) All firms offer both products. We solve the game with backward induction.

5.1 Bertrand Oligopoly

Case (1) All firms offer only low quality product

In this case, there is no product differentiation and thus all firms charge the marginal cost c_L. The profit for each firm is zero.

Case (2) All firms offer only high quality product

Similarly to Case (1), all firms charge the marginal cost c_H and earn zero profit.

Case (3) One firm offers only low quality product and two firms offer only high quality product

Two firms offering high quality product charge the marginal cost c_H. One firm offering low quality product follows Case (5) in duopoly described in the previous section. Therefore, the firm offering low quality product charges $p_L^* = \frac{c_L+c_H}{2} > c_L$ and earns $\pi_V = \frac{1}{4\bar{\rho}}(c_H - c_L)^2$ if $\bar{\rho} \geq \frac{c_H-c_L}{2}$, and charges $p_L^* = c_H - \bar{\rho}$ and earns $\pi_V = c_H - c_L - \bar{\rho}$ if $\bar{\rho} < \frac{c_H-c_L}{2}$.

Case (4) One firm offers only high quality product and two firms offer only low quality product

Two firms offering low quality product charge the marginal cost c_L. One firm offering high quality product follows Case (4) in duopoly. Therefore, the firm offering high quality product charges $p_H^* = \frac{c_L+c_H+\bar{\rho}}{2}$ if $\bar{\rho} \geq c_H - c_L$, and $p_H^* = c_H$ if $\bar{\rho} < c_H - c_L$. This firm's profit for the first case is $\pi_V = \frac{1}{4\bar{\rho}}(\bar{\rho} - (c_H - c_L))^2$ and for the second case is 0.

Case (5) One firm offers only low quality product, one firm offers only high quality product, and the last firm offers both products

Two firms compete in low quality product market and two firms compete in high quality product market, thus all firms charge the marginal costs and earn zero profit.

Case (6) One firm offers only low quality product and two firms offer both products

Three firms compete in low quality product market and two firms compete in high quality product market, thus all firms charge the marginal costs and earn zero profit. .

Case (7) One firm offers only high quality product and two firms offer both products

Two firms compete in low quality product market and three firms compete in high quality product market, thus all firms charge the marginal costs and earn zero profit.

Case (8) Two firms offer only low quality product and one firm offers both products

Similarly to Case (4), two firms offering low quality product charge the marginal cost c_L and one firm offering both charges $p_H^* = \frac{c_L+c_H+\bar{\rho}}{2}$ if $\bar{\rho} \geq c_H - c_L$, and $p_H^* = c_H$ if $\bar{\rho} < c_H - c_L$. This firm's profit for the first case is $\pi_V^B = \frac{1}{4\bar{\rho}}(\bar{\rho} - (c_H - c_L))^2$ and for the second case is 0.

Case (9) Two firms offer only high quality and one firm offers both products

Similarly to Case (3), two firms offering high quality product charge the marginal cost c_H while one firm offering both charges $p_L^* = \frac{c_L+c_H}{2} > c_L$ and earns $\pi_V = \frac{1}{4\bar{\rho}}(c_H - c_L)^2$ if $\bar{\rho} \geq \frac{c_H-c_L}{2}$, and charges $p_L^* = c_H - \bar{\rho}$ and earns $\pi_V = c_H - c_L - \bar{\rho}$ if $\bar{\rho} < \frac{c_H-c_L}{2}$.

Case (10) All firms offer both products

All firms charge the marginal costs and earn zero profit.

5.2 Product Choice

Given the results from the second stage, we can construct three-player payoff matrices of product choices. Table 5, 6, and 7 shows the payoff matrix of the game with $\bar{\rho} < \frac{c_H - c_L}{2}$. The numbers in each cell show π^A, π^B, π^C respectively. Table 5 shows the case where firm C is choosing only low quality product, Table 6 shows the case where firm C is choosing only high quality product, and table 7 shows the case where firm C is choosing both products.

The best response for each player is shown as bar under the payoff. When $\bar{\rho} < \frac{c_H - c_L}{2}$, every strategy profile except for all firms choosing high quality product only is an equilibrium of the game. Only the firm selling low quality product alone is earning positive profits. This is consistent with duopoly model in the previous section.

Table 8, 9, and 10 show the case with $\frac{c_H - c_L}{2} \leq \bar{\rho} < c_H - c_L$. When $\frac{c_H - c_L}{2} \leq \bar{\rho} < c_H - c_L$, the results are very different from duopoly. In duopoly, equilibria were either market segregation (one firm chooses only low quality product and the other firm chooses only high quality product) or standard Bertrand competition (both firms choose both products). In three-firm oligopoly, any combination of choices can be an equilibrium, except for all firms choosing only high quality product. This is because with three firms, they cannot peacefully segregate the market and thus they earn zero profit for most of the cases. It is also important to note that only the firm selling low quality product alone can earn positive profits if $\bar{\rho} < c_H - c_L$.

Table 11, 12, and 13 show the case with $c_H - c_L \leq \bar{\rho}$. The analysis follows the previous two cases, but in this case "all firms choosing only low quality product" is no longer an equilibrium, because switching to high quality product (or both products) can provide positive profit for the firm. This is because consumers are willing to pay bigger premium for high quality product. Only the firm selling low quality product alone or the firm selling high quality product alone can earn positive profit in this case.

This analysis tells us that in three-firm oligopoly, various market structures can be achieved as an equilibrium. For example, one firm is choosing only low quality product, on firm is choosing only high quality product, and the last one is choosing both products in US baby food market. This is an equilibrium for any value of $\bar{\rho}$ in our model.

6 Conclusion

In this paper, we have analyzed versioning as firms' strategy in both monopoly and oligopoly with two versions of a product: high quality product with high marginal cost and low quality product with low marginal cost. We have shown that a monopolist chooses to provide only low quality product if consumers are not willing to pay large premium to switch to high quality product. Otherwise, it is strictly better for a monopolist to provide both products. The monopolist never chooses to offer only high quality product.

In oligopoly, all firms choosing high quality product is never an equilibrium. In oligopoly, with either two-firm or three-firm, any other combination of product choices can be an equilibrium if the premium consumers are willing to pay is smaller than the half of the difference in marginal costs. If the highest premium consumers are willing to pay for high quality product is greater than the half of the difference in marginal costs, there are only two types of equilibria in duopoly. The first one is that one firm chooses only low quality product and the other firm chooses only high quality product. In this equilibrium, firms peacefully share the market and both firms earn positive profits. The second one is that both firms choose both products and earn zero profit. In three-firm oligopoly, peaceful market segregation does not occur so any combination of product choices except for all firms choosing only high quality product can be an equilibrium. If the premium for high quality product is sufficiently large, all firms choosing only low quality product will no longer be an equilibrium. Our result explains many real-life markets with three firms.

In monopoly, versioning strictly increases the profit if the premium consumers are willing to pay is sufficiently large. In oligopoly, versioning does not necessarily increase the firms' profits. For example, if other firms are choosing only high quality product, choosing only low quality product gives exactly the same profit as choosing both products. However, adding versioning as a strategy choice for each firm gives us the explanation of the composition of many real world markets.

Possible extensions of the model include introducing fixed costs such as extra facility or research fund for high quality product, increasing the types of the products, and increasing the number of firms. In three-firm oligopoly, at least one market has two firms competing, so they charge the marginal cost and earns zero profit. We may relax the assumption on the number of product choices to see if it is possible for three firms to segregate the market and each earns positive profit. If we keep the number of product choices to two, increasing the number of firms does not change our result.

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7 Appendix

In this section, we present the payoff matrices for three-firm oligopoly. Table 5, 6, and 7 show the case with $\bar{\rho} < \frac{c_H - c_L}{2}$. Table 8, 9, and 10 show the case with $\frac{c_H - c_L}{2} \le \bar{\rho} < c_H - c_L$. Table 11, 12, and 13 show the case with $c_H - c_L \le \bar{\rho}$.

		Firm B		
		Low quality only	High quality only	Both products
Firm A	Low	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{0}$
	High	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{c_H - c_L - \bar{\rho}}$	$\underline{0},\ \underline{0},\ \underline{0}$
	Both	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{0}$

Table 5: The payoff matrix when firm C is choosing L

		Firm B		
		Low quality only	High quality only	Both products
Firm A	Low	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{c_H - c_L - \bar{\rho}},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{0}$
	High	$\underline{0},\ \underline{c_H - c_L - \bar{\rho}},\ \underline{0}$	$0,\ 0,\ 0$	$\underline{0},\ \underline{c_H - c_L - \bar{\rho}},\ \underline{0}$
	Both	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{c_H - c_L - \bar{\rho}},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{0}$

Table 6: The payoff matrix when firm C is choosing H

		Firm B		
		Low quality only	High quality only	Both products
Firm A	Low	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{0}$
	High	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{c_H - c_L - \bar{\rho}}$	$\underline{0},\ \underline{0},\ \underline{0}$
	Both	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{0}$

Table 7: The payoff matrix when firm C is choosing B

		Firm B		
		Low quality only	High quality only	Both products
Firm A	Low	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{0}$
	High	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{\frac{(c_H - c_L)^2}{4\bar{\rho}}}$	$\underline{0},\ \underline{0},\ \underline{0}$
	Both	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{0}$	$\underline{0},\ \underline{0},\ \underline{0}$

Table 8: The payoff matrix when firm C is choosing L

Firm B

Firm A	Low quality only	High quality only	Both products
Low	$\underline{0}$, $\underline{0}$, $\underline{0}$	$\underline{\frac{(c_H-c_L)^2}{4\bar{\rho}}}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{0}$
High	$\underline{0}$, $\underline{\frac{(c_H-c_L)^2}{4\bar{\rho}}}$, $\underline{0}$	0, 0, 0	$\underline{0}$, $\underline{\frac{(c_H-c_L)^2}{4\bar{\rho}}}$, $\underline{0}$
Both	$\underline{0}$, $\underline{0}$, $\underline{0}$	$\underline{\frac{(c_H-c_L)^2}{4\bar{\rho}}}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{0}$

Table 9: The payoff matrix when firm C is choosing H

Firm B

Firm A	Low quality only	High quality only	Both products
Low	$\underline{0}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{0}$
High	$\underline{0}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{\frac{(c_H-c_L)^2}{4\bar{\rho}}}$	$\underline{0}$, $\underline{0}$, $\underline{0}$
Both	$\underline{0}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{0}$

Table 10: The payoff matrix when firm C is choosing B

Firm B

Firm A	Low quality only	High quality only	Both products
Low	0, 0, 0	$\underline{0}$, $\underline{\frac{(\bar{\rho}-c_H+c_L)^2}{4\bar{\rho}}}$, $\underline{0}$	$\underline{0}$, $\underline{\frac{(\bar{\rho}-c_H+c_L)^2}{4\bar{\rho}}}$, $\underline{0}$
High	$\underline{\frac{(\bar{\rho}-c_H+c_L)^2}{4\bar{\rho}}}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{\frac{(c_H-c_L)^2}{4\bar{\rho}}}$	$\underline{0}$, $\underline{0}$, $\underline{0}$
Both	$\underline{\frac{(\bar{\rho}-c_H+c_L)^2}{4\bar{\rho}}}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{0}$

Table 11: The payoff matrix when firm C is choosing L

		Firm B		
		Low quality only	High quality only	Both products
	Low	$\underline{0}$, $\underline{0}$, $\underline{\frac{(\bar{\rho}-c_H+c_L)^2}{4\bar{\rho}}}$	$\underline{\frac{(c_H-c_L)^2}{4\bar{\rho}}}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{0}$
Firm A	High	$\underline{0}$, $\underline{\frac{(c_H-c_L)^2}{4\bar{\rho}}}$, $\underline{0}$	0, 0, 0	$\underline{0}$, $\underline{\frac{(c_H-c_L)^2}{4\bar{\rho}}}$, $\underline{0}$
	Both	$\underline{0}$, $\underline{0}$, $\underline{0}$	$\underline{\frac{(c_H-c_L)^2}{4\bar{\rho}}}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{0}$

Table 12: The payoff matrix when firm C is choosing H

		Firm B		
		Low quality only	High quality only	Both products
	Low	$\underline{0}$, $\underline{0}$, $\underline{\frac{(\bar{\rho}-c_H+c_L)^2}{4\bar{\rho}}}$	$\underline{0}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{0}$
Firm A	High	$\underline{0}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{\frac{(c_H-c_L)^2}{4\bar{\rho}}}$	$\underline{0}$, $\underline{0}$, $\underline{0}$
	Both	$\underline{0}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{0}$	$\underline{0}$, $\underline{0}$, $\underline{0}$

Table 13: The payoff matrix when firm C is choosing B