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Deriving Reference-Dependent Utility Function from Pareto or Lognormal Income Distribution: The Role of Migration in Evolution

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Deriving Reference-Dependent Utility Function from Pareto or Lognormal Income Distribution: The Role of Migration in Evolution

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Abstract

This study demonstrates that reference-dependent utility naturally follows from an evolutionary process of the individuals whose income distribution is Pareto or lognormal; each distribution, which has been commonly observed in the real economy, results in a function of gain relative to the reference point. A simulation analysis, where the dependency actually arises through migration, is carried out to illustrate the process.

Keywords:
Pareto distribution, lognormal distribution, reference-dependent utility, migration, evolutionary psychology

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1. Introduction

In most economic analyses, the (direct or indirect) utility of a household or an individual is simply assumed to be a function of the absolute level of consumption or income. However, it has been empirically well known (and theoretically often assumed,) since Veblen (1899) and Duesenberry (1949), that the utility is essentially relative and there exists any reference point; the discussions proposed by Sen (1966) and Stigler and Becker (1977) as well as the findings of Easterlin (1973) and Kahneman and Tversky (1979) have been along those lines.

Nevertheless, the reference-dependency is still regarded as a kind of anomaly and seldom used or assumed in the practical policy analyses such as that of spatial economics. This study starts from two conventional income distributions, both of which have been widely recognized and put to use, and intends to derive reference-dependent utility functions as their natural *outcomes*. The first distribution is that of Pareto (1896), whose structure has been explored by Champernowne (1953) and his successors. The second is the lognormal distribution, which was first used to explain the distribution of income or firm size by Gibrat (1932), developed by Aitchison and Brown (1957), interpreted with a stochastic model by Pestieau and Possen (1979), empirically examined (arguing its applicability to income and consumption distributions) by Battistin et al. (2009), and reinvestigated by Akhundjanov and Toda (2020). Moreover, some argue the conditions in which either or both of the distributions characterize the actual situation such as Aoyama et al. (2004) and Botazzi (2009).[1]

While most theoretical works consider that people's preference or utility function is simply given and the income distribution is somewhat a result of their optimizing behavior, this study does the opposite; utility function, or a function which at least describes the migration behavior of the people, arises from the income distribution through evolution. Even Darwin (1859) has already mentioned the biological evolution of instinct in addition to his discussions on those of physical organs. The developments in evolutionary psychology such as Hamilton (1964), Trivers (1971), Dawkins (1976), and Cosmides and Tooby (1992) have further characterized the role of adaptation in the formation of psychological mechanisms. The works in evolutionary game theory, elaborated by Smith and Price (1973) and their successors, have formally modeled the process (see Vincent and Brown (2005) and Nowak (2006) for their summary.) Moreover,

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[1] It is well known that, in many actual cases, the Pareto distribution is valid around the domain of higher income while the lognormal fits around the other parts. Moreover, some empirical researches find deviations particularly around the both tails of the distribution. In search of better specification, Salem and Mount (1974) applied the gamma distribution, Singh and Maddala (1976) and Dagum (1977) introduced their own types, Kloek and van Dijk (1978) examined the fitness of the generalized gamma, the log t and others, and McDonald (1984) used the generalized beta, along with various propositions, as has been summarized by Kleiber and Kotz (2003). However, this study sticks to the traditional two distributions as the first attempt to propose the basic system.

some studies in behavioral economics have already applied similar idea to their models. Cole et al. (1992) has taken matching, capital accumulation, and bequest into account in formulating the preference. McDermott et al. (2008) has characterized the evolutionary origin of the attitude toward risk in prospect theory. Apesteguia and Ballester (2009) has investigated the existence of the reference dependency. De Fraja (2009) has argued that the origin of human utility is related to the courtship behavior. And Apicella et al. (2014) has empirically shown the evolutionary origins of the endowment effect.

However, as far as the author is aware, there are few studies that have formally argued the origin of the reference-dependent utility based on well-known income distributions along with the migration behavior. This study explores its possibility using simple models.[2]

The rest of the paper is organized as follows: section 2 presents the model based on Pareto distribution and the corresponding outcomes. Section 3 investigates the case of

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[2] In the fields of economics without a consideration for space or migration, the relationship between utility functions and probability (distributions) has been investigated in many ways for decades. The works such as Mosteller and Nogee (1951) and Anscome and Aumann (1963), conducted in prior to Kahneman and Tversky (1979), define the utility directly over probability and the amounts consumed (and estimate its parameters.) Another type of study, such as Charles-Cadogan (2018), which derives the reference-dependent preference taking the random growth of personal income into account, contains some probability variables in it. However, the study of this paper is none of the aforementioned types. The distributions are of the actual income over people but not necessarily regarded as "probability" distributions. And the notion of probability is, in effect, used only in calculating the survival/reproduction rate in evolution.

lognormal distribution. Section 4 exhibits a numerical example of evolution. Section 5 concludes.

2. The Model Based on Pareto Distribution

2.1 A Simple Setting

Suppose that there are geographical regions (or groups of people) $i = 1, 2, \cdots, I$. The total population of region i in period t is n_{it}. Within each region, the physical ability (or income) y differs among the people and conforms to the Pareto distribution. The cumulative distribution function is:

$$F_{it} = 1 - \left(\frac{\underline{y_i}}{y}\right)^{\alpha_{it}}, \tag{1}$$

where $\underline{y_i}$ is a scale parameter, which represents the lowest value of y in region i, and α_{it} is a shape parameter in region i in period t.[3] Then, the corresponding density function is,

$$f_{it} = \alpha_{it} y^{-\alpha_{it}-1} \underline{y_i}^{\alpha_{it}}. \tag{2}$$

The population of those with ability y in region i is, multiplying by n_{it},

$$n_{it} f_{it} = n_{it} \alpha_{it} y^{-\alpha_{it}-1} \underline{y_i}^{\alpha_{it}}. \tag{3}$$

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[3] In this subsection, consider the case in which only the shape parameter changes over time. The structure being kept, one can still say the Pareto distribution characterizes the economy.

In the next period, as α_{it} and total population n_{it} change to $\alpha_{i(t+1)}$ and $n_{i(t+1)}$, respectively, the population of those with y in region i becomes:

$$n_{i(t+1)}f_{i(t+1)} = n_{it}\alpha_{i(t+1)}y^{-\alpha_{i(t+1)}-1}\underline{y}_i^{\alpha_{i(t+1)}}. \tag{4}$$

If each period t represents the corresponding "generation" of the people and y is perfectly inherited or socially fixed over generations,[4] an individual in period (generation) t will recognize the number his/her descendants in period $t+1$, or the rate of survival/reproduction:[5]

$$\begin{aligned} v_{it} &= \frac{n_{i(t+1)}f_{i(t+1)}}{n_{it}f_{it}} = \frac{n_{i(t+1)}\alpha_{i(t+1)}}{n_{it}\alpha_{it}}y^{\alpha_{it}-\alpha_{i(t+1)}}\underline{y}_i^{-(\alpha_{it}-\alpha_{i(t+1)})} \\ &= G_{it}\left(\frac{y}{\underline{y}_i}\right)^{A_i}, \end{aligned} \tag{5}$$

where $A_i \equiv \alpha_{it} - \alpha_{i(t+1)}$, which is assumed to be constant over time, and $G_{it} \equiv n_{i(t+1)}\,\alpha_{i(t+1)}/(n_{it}\alpha_{it})$.

If $A_i > 0$ and $y > 1$, v_{it} is positively associated with $y/\underline{y}_i$; one's survival/reproduction rate depends on his/her ability relative to the reference point in each region. One possible mechanism underlying (5) is the chance of mating in a group. Actually, most theoretical (as well as empirical) studies, such as Becker (1973 and 1974),

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[4] For the general intergenerational immobility, refer to Corak (2004), for example.

[5] One can think that, if y is interpreted as the "income" but not the "ability," it may also grow in sufficient length of time. However, in that case, by raising all ys and $\underline{y}_i$s at the same rate, the system does not essentially change. The fact is in accordance with the arguments of the Easterlin's paradox. To make the model as simple as possible, the factor is omitted on this paper.

Mortensen (1988), and Browning et al. (2014), suggest that mating is typically "positive assortative" and in that case, some people with relatively low ability can be left unmated when there are (opportunity) costs of searching or being mated (e.g. married.)[6]

If the behavior of an individual (e.g. migration) is such that he/she seeks higher v_{it}, it works as if it were an (indirect) utility function characterizing his/ migration behavior.[7] Note that it has a typical reference-dependent form.[8] Further behavioral background and an example of the process of utility formation are to be presented in Section 4.

2.2 Extension

Next, consider the case in which $\underline{y_i}$ may change over time, in addition to α_{it}. A simple

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[6] Among the other significant literatures on matching and its stability are Gale and Shapley (1962), Roth and Sotomayeor (1990), Eeckout (2000), and Unayama (2014). One may note that, nowadays, the crude reproduction rate seems to be negatively related to the income level in most countries as Birg (2002) argues and calls the phenomenon "demo-economic paradox." Guinnane (2011) has shown that, before the 19th century, the elasticity of fertility with respect to income had been positive but then the sign switched in Europe and North America. Alfani and García Montero (2022) indicates, even the wealth distribution in the 13th century can be characterized well by the lognormal distribution. Moreover, Myrskylä et al. (2009) has found that, in highly developed countries (in terms of human development index,) further development can reverse the declining trend in fertility. Taking them into account, this study illustrates the system in which the evolution process is stable (i.e. $A_i > 0$.)

[7] As Dufwenberg et al. (2011) summarizes, a direct utility function is more difficult to handle than its indirect counterpart in the context of reference-dependency because the former is defined on consumption bundle; for example, usually a person has envy at others' opportunity of consumption but not the (final) consumption level of each good such as a food he/she dislikes. As a result, the latter (or one-good model) is often used, being more convenient to describe the reference-dependent situations.

[8] One may notice that this form is a family of those presented by Charles-Cadogan (2018) or Yonemoto (2021).

result is obtained when:[9]

$$\underline{y}_{i(t+1)} = \underline{y}_{it}{}^{\beta_{i(t+1)}}, \tag{6}$$

where $\beta_{i(t+1)} \geq 1$. Then, (5) can be rewritten as:

$$\begin{aligned} v_{it} &= \frac{n_{i(t+1)} f_{i(t+1)}}{n_{it} f_{it}} = \frac{n_{i(t+1)} \alpha_{i(t+1)}}{n_{it} \alpha_{it}} y^{\alpha_{it} - \alpha_{i(t+1)}} \underline{y}_{it}{}^{\beta_{i(t+1)} \alpha_{i(t+1)} - \alpha_{it}} \\ &= G_{it} y^{(\beta_{i(t+1)} - 1)\alpha_{i(t+1)}} \left(\frac{y}{\underline{y}_{it}} \right)^{\alpha_{it} - \beta_{i(t+1)} \alpha_{i(t+1)}}. \end{aligned} \tag{5'}$$

That is, y is evaluated by the absolute part as well as the relative one. Instead of deriving from (6), one can directly assume (5'). For example, it is more likely that the absolute income (ability) is somewhat important in survival while the relative position may matter in mating. Keeping consistency with (2), one may write:

$$v_{it} = G_{it} y^{(1-\eta_{i(t+1)})A_i} \left(\frac{y}{\underline{y}_{it}} \right)^{\eta_{i(t+1)} A_i}, \tag{7}$$

where $0 \leq \eta_{i(t+1)} \leq 1$ corresponds to $\left(\alpha_{it} - \beta_{i(t+1)} \alpha_{i(t+1)}\right)/A_i$ in (5'). In the following case in particular,

$$\beta_{it} = (1-\eta) \frac{A_i}{\alpha_{it}} + 1, \tag{8}$$

η is a constant and (7) can be described even simpler:

$$v_{it} = G_{it} y^{(1-\eta)A_i} \left(\frac{y}{\underline{y}_{it}} \right)^{\eta A_i} = G_{it} \left(\frac{y}{\underline{y}_{it}{}^{\eta}} \right)^{A_i}, \tag{5''}$$

which is to be used in the simulation in section 4.

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[9] In the context of the arguments of footnote (5), this case corresponds to the situation in which $\underline{y}_i$ grows relative to y, when all ys and $\underline{y}_i$s are rising at the same rate.

3. The Model Based on Lognormal Distribution

3.1 A Simple Setting

If the income is distributed lognormal, the density function is,

$$\tilde{f}_{it} = \frac{1}{\sqrt{2\pi} \cdot \sigma y} \exp\left(-\frac{(\ln y - \ln m_{it})^2}{2\sigma^2}\right). \tag{9}$$

where σ and m_{it} are parameters.[10] Note that m_{it} corresponds to the median of the lognormal distribution. Then, the probability of survival/reproduction is,

$$\begin{aligned}\tilde{v}_{it} &= \frac{n_{i(t+1)}\tilde{f}_{i(t+1)}}{n_{it}\tilde{f}_{it}} = \frac{n_{i(t+1)}}{n_{it}} \exp\left(\frac{-\left(\ln y - \ln m_{i(t+1)}\right)^2 + (\ln y - \ln m_{it})^2}{2\sigma^2}\right) \\ &= \frac{n_{i(t+1)}}{n_{it}} \exp\left(\frac{\gamma_i(2\ln y - 2\ln m_{it} - \gamma_i)}{2\sigma^2}\right),\end{aligned} \tag{10}$$

where $\gamma_i \equiv \ln m_{i(t+1)} - \ln m_{it}$ for m_{it} growing at a constant rate. (10) can be further rewritten as:

$$\begin{aligned}&\tilde{v}_{it} = \tilde{G}_{it} \left(\frac{y}{m_{it}}\right)^{\tilde{A}_i}, \\ &\text{where} \quad \tilde{A}_i \equiv \frac{\gamma_i}{\sigma^2} \quad \text{and} \quad \tilde{G}_{it} \equiv \frac{n_{i(t+1)}}{n_{it}} \exp\left(-\frac{\gamma_i^2}{2\sigma^2}\right).\end{aligned} \tag{11}$$

(11) is reference-dependent and similar to (5) in the last section. In this lognormal case, the reference point is the median (m_{it}). The other parameter σ, which constitutes $\tilde{A}_i$ and $\tilde{G}_{it}$, is associated with the variance of the distribution.

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[10] In this subsection, suppose only m_{it} changes over time

3.2 Extension

Now, let us see if it is possible for y and m_{it} to have different degrees such as in subsection 2.2. Similar to (7), one can directly assume that there is another factor representing the absolute effect:

$$\tilde{v}_{it} = \tilde{G}_{it} y^{(1-\tilde{\eta}_{i(t+1)})\tilde{A}_i} \left(\frac{y}{m_{it}}\right)^{\tilde{\eta}_{i(t+1)}\tilde{A}_i} = \breve{G}_{it}\left(\frac{y}{m_{it}^{\tilde{\eta}_{i(t+1)}}}\right)^{\breve{A}_i}, \tag{12}$$

where $0 \leq \tilde{\eta}_{i(t+1)} \leq 1$. Actually, (12) is obtained when the change in m_{it} is described more generally as follows:

$$m_{i(t+1)} = e^{\tilde{\gamma}_{it}} m_{it}^{\tilde{\beta}_{it}}, \tag{13}$$

where $\tilde{\beta} \leq 1$, $\tilde{\gamma}_{it} > 0$.[11] Then, (10) is rewritten as:

$$\tilde{v}_{it} = \frac{n_{i(t+1)}}{n_{it}} \exp\left(\frac{\left[\left(\tilde{\beta}_{it} - 1\right)\ln m_{it} + \tilde{\gamma}_{it}\right]\left[2\ln y - \left(\tilde{\beta}_{it} + 1\right)\ln m_{it} - \tilde{\gamma}_{it}\right]}{2\sigma^2}\right). \tag{10'}$$

(10') can be expressed as:

$$\tilde{v}_{it} = \breve{G}_{it}\left(\frac{y}{m_{it}^{\tilde{\eta}_{i(t+1)}}}\right)^{\breve{A}_{it}}, \tag{11'}$$

where

$$\breve{A}_{it} \equiv \frac{\left(\tilde{\beta}_{it} - 1\right)\ln m_{it} + \tilde{\gamma}_{it}}{\sigma^2},$$
$$\breve{G}_{it} \equiv \frac{n_{i(t+1)}}{n_{it}} \exp\left(-\frac{\tilde{\gamma}_{it}\left[\left(\tilde{\beta}_{it} - 1\right)\ln m_{it} + \tilde{\gamma}_{it}\right]}{2\sigma^2}\right), \quad \text{and} \quad \breve{\eta}_{i(t+1)} \equiv \frac{\tilde{\beta}_{it} + 1}{2}. \tag{14}$$

[11] Because it requires smaller $\tilde{\beta}$, while $e^{\tilde{\gamma}_{it}}$ may grow, the interpretation of (13) is slightly harder than in the case of (6). One can also consider the case in which $\tilde{\beta} > 1$ but it may result in $\breve{\eta} > 1$ in (11') and the function may depend *too much* on the reference point.

In the following particular case,

$$(\tilde{\beta}_{it} - 1)\ln m_{it} + \tilde{\gamma}_{it} = \delta_i, \tag{15}$$

$\check{A}_{it}$ in (11') becomes constant:

$$\tilde{v}_{it} = \check{G}_{it}\left(\frac{y}{m_{it}^{\check{\eta}_{i(t+1)}}}\right)^{\check{A}_i}, \tag{11''}$$

where

$$\check{A}_i \equiv \frac{\delta_i}{\sigma^2}, \quad \check{G}_{it} \equiv \frac{n_{i(t+1)}}{n_{it}}\exp\left(-\frac{\tilde{\gamma}_{it}\delta_i}{2\sigma^2}\right). \tag{16}$$

Moreover, if $\tilde{\beta}_{it} = \tilde{\beta}$, $\check{\eta}_{i(t+1)}$ also becomes constant:

$$\tilde{v}_{it} = \check{G}_{it}\left(\frac{y}{m_{it}^{\check{\eta}}}\right)^{\check{A}_i}, \tag{11'''}$$

(11''') is similar to (5'') in subsection 2.2; it differs only in the definition of the reference point.

4. Characterization of the Evolution

4.1 Separate Locations

Each of the functions derived in the preceding sections represents the corresponding probability of survival/reproduction but is not always called a "utility" function in economic theory. Nevertheless, if any people's behavior, such as migration, is consistent with it (e.g. they have a "correct" expectation on the parameters,) they or their descendants are more likely to be sustained in the successive periods. As a result, in the long run (through an evolutionary process,) the function is expected to be actually representing the people's behavior; in later periods, given a "utility" function that have been already formed, each individual may act so as to achieve higher its value or the level of his/her "satisfaction."[12]

Illustrate the process with a simple example. Suppose the actual of rate of survival/reproduction is characterized by (5") in subsection 2.2. Consider the case in which most people do not know the true η but have a priori guess $\breve{\eta} \in \left[\underline{\eta}, \overline{\eta}\right]$. Then, the corresponding evaluation is:

$$\breve{v}_{it} = G_{it} y^{A_i} \underline{y}_{it}{}^{-\breve{\eta} A_i}, \tag{17}$$

Suppose that there are only two regions $(i = 1, 2)$ and $A_2 > A_1$ without loss of generality. The perceived difference in utility between the two regions is:

$$\breve{v}_{2t} - \breve{v}_{1t} = G_{2t} y^{A_2} \underline{y}_{2t}{}^{-\breve{\eta} A_2} - G_{1t} y^{A_1} \underline{y}_{1t}{}^{-\breve{\eta} A_1}. \tag{18}$$

With no error terms in (18), for each level of $\breve{\eta}$, $y = \breve{y}_{\breve{\eta}}$ that makes the difference zero, is derived:[13]

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[12] Regional models those assume reference-dependency include Yonemoto (2021), for example.

[13] With error terms, people's location choice is described by a discrete-choice model and they are divided into the regions more smoothly. Then, as has been argued in Yonemoto (2023), the resulting distribution (of each region) is also approximated well by the Pareto (or lognormal) distribution while the analysis is made more complex. In the case of the complete separation described by (19), the distribution looks truncated within each region while the one

$$\breve{y}_{\hbar} = \left(\frac{G_{1t}}{G_{2t}}\right)^{\frac{1}{A_2 - A_1}} \left(\frac{\underline{y_{2t}}^{A_2}}{\underline{y_{1t}}^{A_1}}\right)^{\frac{\breve{\eta}}{A_2 - A_1}}. \tag{19}$$

Differentiating (16) with respect to y,

$$\frac{d(\breve{v}_{2t} - \breve{v}_{1t})}{dy} = A_2 G_{2t} y^{A_2 - 1} \underline{y_{2t}}^{-\breve{\eta} A_2} - A_1 G_{1t} y^{A_1 - 1} \underline{y_{1t}}^{-\breve{\eta} A_1}. \tag{20}$$

Evaluating at $\breve{v}_{2t} = \breve{v}_{1t}$,

$$\left.\frac{d(\breve{v}_{2t} - \breve{v}_{1t})}{dy}\right|_{\breve{v}_{2t} = \breve{v}_{1t}} = \frac{A_2 - A_1}{y} \breve{v}_{1t} > 0. \tag{21}$$

That is, the people tend to locate separately according to their ability (income) levels. As a result, it is expected that $\underline{y_{2t}} > \underline{y_{1t}}$.[14] By (17), each $\breve{v}_{it}$ is a decreasing function of $\breve{\eta}$ (for $\underline{y_{it}} > 1$.) Also, differentiating (18) with respect to $\breve{\eta}$,

$$\frac{d(\breve{v}_{2t} - \breve{v}_{1t})}{d\breve{\eta}} = -A_2 G_{2t} y^{A_2} \underline{y_{2t}}^{-\breve{\eta} A_2} \ln \underline{y_{2t}} + A_1 G_{1t} y^{A_1} \underline{y_{1t}}^{-\breve{\eta} A_1} \ln \underline{y_{1t}}. \tag{22}$$

Again, evaluating at $\breve{v}_{2t} - \breve{v}_{1t}$,

$$\left.\frac{d(\breve{v}_{2t} - \breve{v}_{1t})}{d\breve{\eta}}\right|_{\breve{v}_{2t} = \breve{v}_{1t}} = (-A_2 + A_1) \breve{v}_{1t} \ln \underline{y_{1t}} < 0. \tag{23}$$

Thus, at least around the vicinity of the intersection of $\breve{v}_{1t}$ and $\breve{v}_{2t}$, an individual with higher $\breve{\eta}$ tends to evaluate region 2 relatively lower.

By (21) and (23), a typical "border" ($\breve{v}_{1t} = \breve{v}_{2t}$) locus can be drawn on a diagram with $y - \breve{\eta}$ axes (see Figure 1.) The individuals in the upper left area of the locus actually

in the entire economy is not far from the original at least in the initial periods.

[14] Actually, then, $\underline{y_{2t}} = \breve{y}_{\hbar}$ can be regarded as a function of people's migration. For simplicity, within this study, people expect that $\underline{y_{2t}}$ is constant or changes (exogenously) in a similar manner to $\underline{y_{1t}}$.

settle down in region 1 while the rest others locate in region 2. Note that $\breve{\eta}$ has a true value η, which correctly makes (17) correspond to the actual survival/reproduction rate (5"); $\breve{y}_{\eta t}$ denotes the corresponding y at the border. The individuals in the shaded areas are actually "wrong" in their location choice. Those in the lower shaded area are in region 2 while it is more appropriate to be in region1; in the upper shaded area, the opposite is the case.

Note that this type of sorting does not occur with a single-region setting; as people have no chance to choose, survival/reproduction occurs automatically without any process of adaptation. In this two-region setting, people reveal their preferences

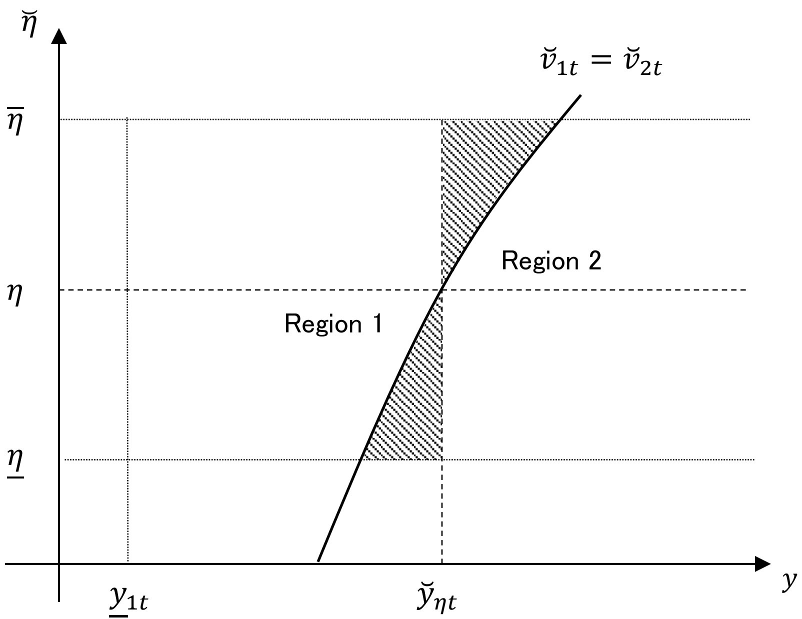

Figure 1: Separate Residential Locations

through migration, causing evolution as a result.

The actual survival/reproduction rate v_{it} of an individual whose subjective assessment of the reference dependency is $\breve{\eta} > (<)\, \eta$ while $y > (<)\, \breve{y}_{\eta t}$ is, because he/she decides to live in region 1 (2,) lower than the one in region 2 (1.) by:

$$|v_{2t} - v_{1t}| = \left| G_{2t} y^{A_2} \underline{y}_{2t}{}^{-\eta A_2} - G_{1t} y^{A_1} \underline{y}_{1t}{}^{-\eta A_1} \right|. \tag{24}$$

As a result, it is harder for those individuals to survive/reproduce than the others, who correctly choose the locations. Thus, in later periods, the share of the individuals whose subjective $\breve{\eta}$ is close to η becomes higher; the locational decisions cause evolution in preference.

4.2 A Numerical Example

Illustrate the first step of the process with a simple example. The values of variables and parameters are set as follows: $y \geq 2.0$ (drawn on the figure up to $y = 20.0$,) $\alpha_{11} = 0.4$, $\alpha_{21} = 0.8$, $A_1 = 0.004$, $A_2 = 0.008$, $\eta = 0.7$ ($0.6 \leq \breve{\eta} \leq 0.8$; uniformly distributed,) and $G_{1t} = G_{2t} = G = 0.99$.

The rate of excess decline, which is (24) divided by the optimal v_{it} (whichever higher of v_{1t} and v_{2t},) is calculated along $\breve{\eta}$ axis as well as that of y (Figures 2 and 3.) Note that the figures are depicted upside down in order to emphasize the actual effect on population. Those with preference around η has higher survival/reproduction rate. Interestingly, when the rate of excess decline is observed along y axis, it has two peaks (or the "bottoms" as they look on the figure,) not around at $\breve{y}_{\eta t}$ but neighboring domains; physically indifferent people are not much affected but mentally "erroneous" people can be.

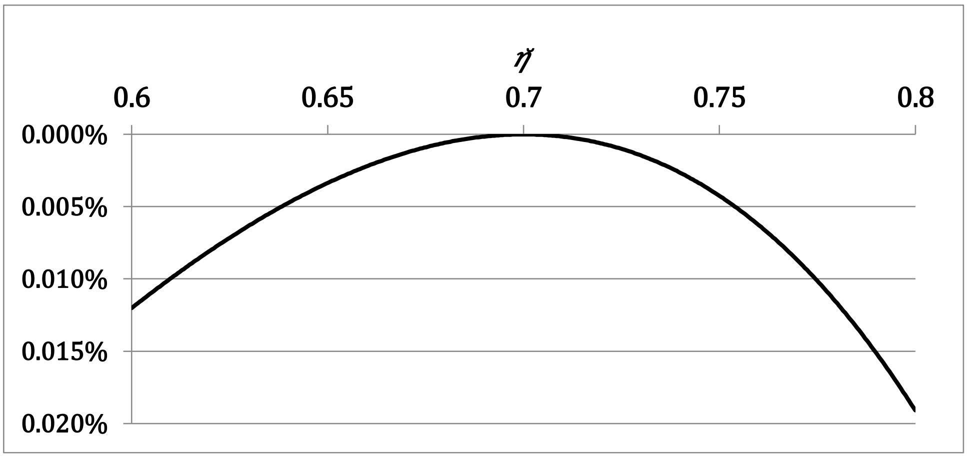

Figure 2: Numerical Example of Excess Decline ($\breve{\eta}$ axis, Flipped Vertically)

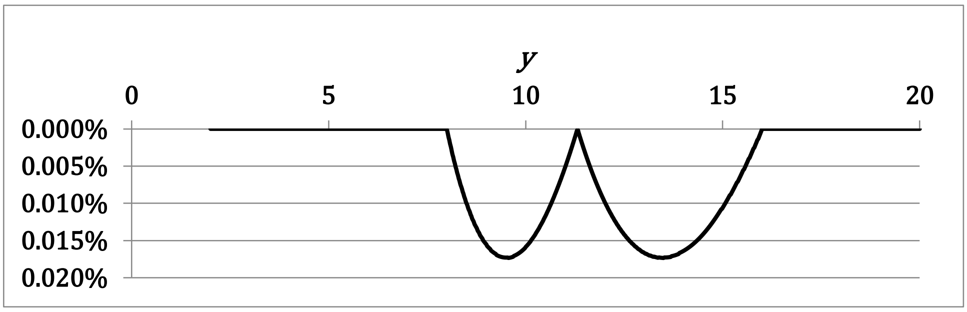

Figure 3: Numerical Example of Excess Decline (y axis, Flipped Vertically)

5. Concluding Remarks

In this study, reference-dependent utility function has been derived consistently from Pareto and lognormal distributions. Also, an example of an evolutionary process, where the dependency actually arises through migration, has been presented. A hypothesis which is familiar to behavioral economists has been associated with one of the most traditional empirical findings in economics.

Note that this study is not intended for an interest on mathematical possibility of modelling or credulous application of Darwinism, as has been criticized by many including Rose and Rose (2000) and Richardson (2007), but a part of efforts to theoretically explore the origin of our preference and migration, which have been conducted for millennia.

There can be extensions and applications, which have not been included in this paper, in many directions. First, elements other than envy (or the relative position to the others) in human preference or society, such as altruism (voluntary cooperation, alms, or the role of agglomeration) can be introduced to the modeling. Second, more general simulation analyses, such as the one with smoother discrete-choice migrations over many regions, might be conducted. Third, a comparison to the preceding studies of the type deriving income distributions from the behaviors of individuals with traditional utility (preference) assumptions would be made. Empirical or experimental studies on the models presented in this paper are also possible.

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References

Aitchison, J., and Brown, J. A. C., 1957. The Lognormal Distribution. Cambridge University Press, Cambridge.

Akhundjanov, S. B., and Toda, A. A., 2020. Is Gibrat's "Economic Inequality" Lognormal?. *Empirical Economics* 59, 2071-2091.

Alfani, G., and García Montero, H., 2022. Wealth Inequality in Pre-Industrial England: A Long-Term View (Late Thirteenth to Sixteenth Centuries). *The Economic History Review* 75, 1314-1348.

Anscombe, F. J., and Aumann, R. J., 1963. A Definition of Subjective Probability. *Annals of Mathematical Statistics* 34, 199-205.

Apesteguia, J., and Ballester, M. A., 2009. A Theory of Reference-dependent Behavior. *Economic Theory* 40, 427–455.

Apicella, C. L., Azevedo, E. M., Christakis, N. A., and Fowler, J. H., 2014. Evolutionary Origins of the Endowment Effect: Evidence from Hunter-gatherers. *American Economic Review* 104(6), 1793-1805.

Aoyama, H., Fujiwara, Y., and Souma, W., 2004. Kinematics and Dynamics of Pareto–Zipf's Law and Gibrat's Law. *Physica A: Statistical Mechanics and Its Applications* 344(1-2), 117-121.

Battistin, E., Blundell, R., and Lewbel, A., 2009. Why is Consumption More Log Normal than Income? Gibrat's Law Revisited. *Journal of Political Economy* 117(6), 1140-1154.

Becker, G. S., 1973. A Theory of Marriage: Part 1. *Journal of Political Economy* 81, 813-846.

Becker, G. S., 1974. A Theory of Marriage: Part 2. *Journal of Political Economy* 82, 11-26.

Birg, H., 2002. Demographic Ageing and Population Decline in 21st Century Germany: Consequences for the Systems of Social Insurance. *Population Bulletin of the United Nations* 44/45, 103–134.

Bottazzi, G., 2009. On the Irreconcilability of Pareto and Gibrat Laws. *Physica A: Statistical Mechanics and its Applications* 388(7), 1133-1136.

Browning, M., Chiappori, P. A., and Weiss, Y., 2014. *Economics of the Family*. Cambridge University Press, Cambridge.

Champernowne, D. G., 1953. A Model of Income Distribution. *The Economic Journal* 63(250), 318-351.

Charles-Cadogan, G., 2018. Losses Loom Larger than Gains and Reference Dependent Preferences in Bernoulli's Utility Function", *Journal of Economic Behavior and Organization* 154, 220-237.

Cole, H. L., Mailath, G. J., and Postlewaite, A., 1992. Social Norms, Savings Behavior, and Growth. *Journal of Political Economy* 100(6), 1092-1125.

Corak, M. (ed.,) 2004. *Generational Income Mobility in North America and Europe*. Cambridge University Press.

Cosmides L., and Tooby, J., 1992. Cognitive Adaptations for Social Exchange. In *the Adapted Mind: Evolutionary Psychology and the Generation of Culture* (Chapter 3, 163-228,) Oxford University Press, New York.

Dagum, C., 1977. A New Model of Personal Income Distribution: Specification and Estimation. *Economie Appliquée* 33, 327-367.

Darwin, C., 1859. *On the Origin of Species*. Murray, London.

Dawkins, R., 1976. *The Selfish Gene*. Oxford University Press, Oxford.

De Fraja, G., 2009. The Origin of Utility: Sexual Selection and Conspicuous Consumption. *Journal of Economic Behavior & Organization* 72, 51–69.

Duesenberry, J., 1949. *Income, Saving and the Theory of Consumer Behavior*. Harvard University Press, Cambridge.

Dufwenberg, M., Heidhues, P., Kirchsteiger, G., Riedel, F., and Sobel, J., 2011. Other-Regarding Preferences in General Equilibrium. *The Review of Economic Studies* 78, 613-639.

Easterlin, R. A., 1974. Does Economic Growth Improve the Human Lot? Some Empirical Evidence. In *Nations and Households in Economic Growth* (89-125,) Academic Press.

Eeckhout, J., 2000. On the Uniqueness of Stable Marriage Matchings. *Economics Letters* 69(1), 1-8.

Gale, D., and L. Shapley, 1962. College Admission and the Stability of Marriage. *American Mathematical Monthly* 69, 9-15.

Gibrat, R., 1931. *Les Inégalités Économiques*. Paris, Librairie du Recueil Sirey.

Guinnane, T. W., 2011. The Historical Fertility Transition: A Guide for Economists. *Journal of Economic Literatu*re 49, 589-614.

Hamilton, W. D., 1964. The Genetical Evolution of Social Behaviour, I and II. *Journal of Theoretical Biology* 7(1), 1-52.

Kahneman, D., and Tversky, A., 1979. Prospect Theory: an Analysis of Decision Under Risk. *Econometrica*, 47(2), 263-291.

Kleiber, C. and Kotz, S., 2003. *Statistical Size Distributions in Economics and Actuarial Sciences*. John Wiley and Sons, Inc.

Kloek, T. and van Dijk, H. K., 1978. Efficient Estimation of Income Distribution Parameters. *Journal of Econometrics* 8, 61-74.

McDermott, R., Fowler, J. H., and Smirnov, O., 2008. On the Evolutionary Origin of Prospect Theory Preferences. *The Journal of Politics* 70(2), 335-350.

Mortensen, D., 1988. Matching: Finding a Partner for Life or Otherwise. *American Journal of Sociology* 94, S215-S240.

Mosteller, F., and Nogee, P., 1951. An Experimental Measurement of Utility", *Journal of Political Economy* 59, 371-404.

Myrskylä, M., Kohler, H. P., and Billari, F. C., 2009. Advances in Development Reverse Fertility Declines. *Nature* 460 (7256), 741-743.

Nowak, M. A., 2006. *Evolutionary Dynamics: Exploring the Equations of Life*. Harvard University Press, Cambridge.

Pareto, V., 1896. Il modo di figurare i fenomenieconomici. *Giornale degli Economisti*.

Pestieau, P., and Possen, U. M., 1982. A Model of Income Distribution. *European Economic Review* 17, 279-294.

Richardson, R. C., 2010. *Evolutionary Psychology as Maladapted Psychology*. MIT press.

Rose, H., and Rose, S., 2010. *Alas Poor Darwin: Arguments Against Evolutionary Psychology*. Random House.

Roth, A., and Sotomayor, M., 1990. *Two-sided Matching. A Study in Game-theoretic Modelling and Analysis*. Cambridge University Press, Cambridge.

Salem, A. B., and Mount, T. D., 1974. A Convenient Descriptive Model of Income Distribution: The Gamma Density. *Econometrica* 42, 1115-1127.

Sen, A., 1966. Labour Allocation in a Cooperative Enterprise. *Review of Economic Studies* 33(4), 361-71.

Singh, S. K., and G. S. Maddala, 1976. A Function for Size Distribution of Income. *Econometrica* 44, 963-970.

Smith, J. M., and Price, G. R., 1973. The Logic of Animal Conflict. *Nature* 246, 15–18.

Stigler, G. J., and G. S. Becker, 1977. De Gustibus Non Est Disputandum. *American Economic Review* 67(2), 76-90.

Trivers, R. L., 1971. The Evolution of Reciprocal Altruism. *Quarterly Review of Biology* 46, 35-57.

Unayama, T., 2014. Female Labor Market, Intra-household Allocation, and Marriage. *RIETI DP* 14-J-048 (in Japanese.)

Veblen, T., 1899. *The Theory of the Leisure Class: An Economic Study in the Evolution of Institutions*. The Macmillan Company, New York.

Vincent, T. L., and Brown, J. S., 2005. *Evolutionary Game Theory, Natural Selection, and Darwinian Dynamics*. Cambridge University Press, Cambridge.

Yonemoto, K., 2021. Reference-dependent Preference and Interregional Migration: Extending the Harris–Todaro Model. *Letters in Spatial and Resource Sciences* 14, 1–10.

Yonemoto, K., 2023. Migration, Reference-Dependency, Income Distribution, and Its Stability. A presented paper in 70th Annual Meeting, North American Meetings of the Regional Science Association International.

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