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Infectious Disease, Dynamic and Static Externalities in Two-Location Model: Theoretical and Simulation Analyses

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Theoretical and Simulation Analyses

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1. Introduction

This study theoretically characterizes the optimal control path of a two-location economy in which production/consumption has a positive externality while infectious disease, such as COVID-19, has a negative (dynamic) externality.

The model is based on Bobashev et al. (2011), Bethune and Korinek (2020), Alvarez et al. (2020), and Eichenbaum et al. (2020), Akamatsu, et al. (2020) and Olivares et al. (2020), which follow the seminal works of Kremack and McKendrick (1927) and Anderson and May (1991), but taking into account the fact that COVID-19 has been mostly transmitted between regions not by migrants but by travelers or commuters, it assumes the residents do not migrate. Instead, it assumes the disease is transmitted through the economic activities of the residents over the regions as well as within each region.

2. The Model

The basic model is described as follows. Consider an economy with two locations: $i = 1, 2$ (e.g., the center and the suburb in a city.) The population of region i is denoted by N_i and assumed to be constant over time. The number of susceptible, infected, and recovered residents in location i in period t are denoted by S_{it}, I_{it}, and R_{it}, respectively. Their changes are affected by the domestic activity levels of the residents of type $Z = S, I, R$, those of the residents who visit the other location (and back home,) and those of the residents who are visiting from the other region, represented by β_{0iit}, β_{1ijt}, and β_{2jit}:

$$\dot{S}_{it} = -\left(\beta_{0iit} I_{it} S_{it} + \beta_{1ijt} I_{jt} S_{it} + \beta_{2jit} I_{jt} S_{it}\right)$$

$$\dot{I}_{it} = \beta_{0iit} I_{it} S_{it} + \beta_{1ijt} I_{jt} S_{it} + \beta_{2jit} I_{jt} S_{it} - \gamma_i I_{it}, \tag{1}$$

$$\dot{R}_{it} = \gamma_i I_{it}$$

where γ_i is the rate of recovery. Also, assume that each activity level is a linear function of the consumption of the corresponding good.

$$\beta_{0iit} = \beta_0 x_{iit}^S x_{iit}^I, \quad \beta_{1ijt} = \beta_1 x_{ijt}^S x_{jjt}^I, \quad \beta_{2jit} = \beta_2 x_{iit}^S x_{jit}^I,$$
$$x_{\cdot\cdot t}^Z \le 1, \quad Z = S, I, R. \tag{2}$$

The utility is characterized by:

$$u_{it}^Z = \begin{cases} u\left(x_{iit}^Z, x_{ijt}^Z;\ X_{it}, X_{jt}\right) & \text{for} \quad Z = S, R, \\ u\left(x_{iit}^Z, x_{ijt}^Z;\ X_{it}, X_{jt}\right) - c & \text{for} \quad Z = I \end{cases}, \tag{3}$$

where production/consumption externality X_{it} is characterized by,

$$X_{it} \equiv \sum_Z \left(\phi_c + \phi_0 Z_{it} x_{iit}^Z + \phi_2 Z_{jt} x_{jit}^Z\right), \quad Z = S, I, R. \tag{4}$$

Specify u as follows:

$$u\left(x_{iit}^Z, x_{ijt}^Z;\ X_{it}, X_{jt}\right) = X_{it}\left(x_{iit}^Z\right)^\sigma + \theta X_{jt}\left(x_{ijt}^Z\right)^\sigma, \tag{5}$$

For the command optimum, the maximand is,

$$V = \int_0^T e^{-\rho t} \sum_i \{S_{it} u(x_{iit}^S, x_{ijt}^S;\ X_{it}, X_{jt}) + I_{it}[u(x_{iit}^I, x_{ijt}^I;\ X_{it}, X_{jt}) - c]$$

$$+R_{it} u(x_{iit}^R, x_{ijt}^R;\ X_{it}, X_{jt})\} dt - \Phi(I_{1T}, I_{2T}), \quad j \neq i. \tag{6}$$

The Hamiltonian is:

$$H = e^{-\rho t}\{(N_1 - I_{1t} - R_{1t}) u(x_{11t}^S, x_{12t}^S;\ X_{1t}, X_{2t})$$

$$+I_{1t}[u(x_{11t}^I, x_{12t}^I;\ X_{1t}, X_{2t}) - c] + R_{1t} u(x_{11t}^R, x_{12t}^R;\ X_{1t}, X_{2t})$$

$$+(N_2 - I_{2t} - R_{2t}) u(x_{22t}^S, x_{21t}^S;\ X_{2t}, X_{1t})$$

$$+I_{2t}[u(x_{22t}^I, x_{21t}^I;\ X_{2t}, X_{1t}) - c] + R_{2t} u(x_{22t}^R, x_{21t}^R;\ X_{2t}, X_{1t})\} \tag{7}$$

$$+\lambda_{1t}^I\{(N_1 - I_{1t} - R_{1t})[\beta_{0\,11t} I_{1t} + (\beta_{1\,12t} + \beta_{2\,21t}) I_{2t}] - \gamma_1 I_{1t}\}$$

$$+\lambda_{2t}^I\{(N_2 - I_{2t} - R_{2t})[\beta_{0\,22t} I_{2t} + (\beta_{1\,21t} + \beta_{2\,12t}) I_{1t}] - \gamma_2 I_{2t}\}$$

$$+\lambda_{1t}^R \gamma_1 I_{1t} + \lambda_{2t}^R \gamma_2 I_{2t}.$$

3. Characterizing the Externalities

3.1 Optimality Conditions

For type S, the optimality conditions are:

$$\frac{\partial H}{\partial x_{iit}^S} = e^{-\rho t} \phi_0 (N_i - I_{it} - R_{it}) \left(\frac{N_i - I_{it} - R_{it}}{X_{it}} + \frac{\sigma}{x_{iit}^S} \right) X_{it} (x_{iit}^S)^\sigma$$

$$+\lambda_{it}^I (\beta_0 x_{iit}^I I_{it} + \beta_2 x_{jit}^I I_{jt})(N_i - I_{it} - R_{it}) \tag{8}$$

$$+\lambda_{jt}^I \beta_1 x_{jit}^I I_{it} (N_j - I_{jt} - R_{jt}) = 0, \quad i = 1, 2,$$

$$\frac{\partial H}{\partial x_{ijt}^S} = e^{-\rho t}\left[(N_i - I_{it} - R_{it})\theta\left(\phi_2 \frac{N_i - I_{it} - R_{it}}{X_{jt}} + \phi_0 \frac{\sigma}{x_{ijt}^S}\right) X_{jt}\left(x_{ijt}^S\right)^\sigma \right.$$

$$\left. + \phi_2\left(N_j - I_{jt} - R_{jt}\right)(N_i - I_{it} - R_{it})\left(x_{jjt}^S\right)^\sigma\right]$$

$$+\lambda_{it}^I \beta_1 x_{jjt}^I I_{jt}(N_i - I_{it} - R_{it}) + \lambda_{jt}^I \beta_2 x_{jjt}^I I_{it}\left(N_j - I_{jt} - R_{jt}\right) = 0,$$

$$i = 1, 2, \quad i \neq j.$$

Also, the ones for the co-state variables are,

$$\begin{aligned} \frac{d\lambda_{it}^I}{dt} &= -\frac{\partial H}{\partial I_{it}} \\ &= -\lambda_{it}^I\left[\beta_0 x_{iit}^S x_{iit}^I (N_i - 2I_{it} - R_{it}) - \left(\beta_1 x_{ijt}^S x_{jjt}^I + \beta_2 x_{iit}^S x_{jit}^I\right) I_{jt} - \gamma_i\right] \\ &\quad -\lambda_{jt}^I\left(\beta_1 x_{jit}^S x_{iit}^I + \beta_2 x_{jjt}^S x_{ijt}^I\right)\left(N_j - I_{jt} - R_{jt}\right) + \lambda_{it}^R \gamma_i + e^{-\rho t} c, \\ \frac{d\lambda_{it}^R}{dt} &= -\frac{\partial H}{\partial R_{it}} = 0. \end{aligned} \tag{9}$$

Transversality conditions are:

$$\begin{aligned} \lambda_{iT}^I &= \frac{\partial \Phi(I_{1T}, I_{2T})}{\partial I_{iT}}, \quad \lambda_{iT}^R = 0, \\ \frac{d\lambda_{it}^R}{dt} &= -\frac{\partial H}{\partial R_{it}} = 0. \end{aligned} \tag{10}$$

Note that, the externality of infectious disease is dynamic, represented by λ_{it}^I and they changes while the production/consumption externalities are essentially static and affect the economy through equations (3).

Uniform Control

In the simple case where the authority can only control the activities of the residents irrespective of their types, (2) is summarized by,

$$\beta_{0iit} = \beta_0(x_{iit})^2,\quad \beta_{1ijt} = \beta_1 x_{ijt}x_{jjt},\quad \beta_{2jit} = \beta_2 x_{iit}x_{jit} \tag{2'}$$

And (4) is rewritten as,

$$X_{it} = \phi_c + \phi_0 N_i x_{iit} + \phi_2 N_j x_{jit} \tag{4'}$$

The maximand is,

$$\begin{aligned} V = \int_0^T e^{-\rho t}\{&N_1[X_{1t}(x_{11t})^\sigma + \theta X_{2t}(x_{12t})^\sigma] - cI_{1t} \\ &+ N_2[X_{2t}(x_{22t})^\sigma + \theta X_{1t}(x_{21t})^\sigma] - cI_{2t}\}dt - \Phi(I_{1T}, I_{2T}) \end{aligned} \tag{6'}$$

(7) is rewritten to be:

$$\begin{aligned} H = e^{-\rho t}\{&N_1[X_{1t}(x_{11t})^\sigma + \theta X_{2t}(x_{12t})^\sigma] - cI_{1t} \\ &+ N_2[X_{2t}(x_{22t})^\sigma + \theta X_{1t}(x_{21t})^\sigma] - cI_{2t}\} \\ &+\lambda_{1t}^I\{[\beta_0(x_{11t})^2 I_{1t} + (\beta_1 x_{12t}x_{22t} + \beta_2 x_{11t}x_{21t})I_{2t}](N_1 - I_{1t} - R_{1t}) \\ &- \gamma_1 I_{1t}\} \\ &+\lambda_{2t}^I\{[\beta_0(x_{22t})^2 I_{2t} + (\beta_1 x_{21t}x_{11t} + \beta_2 x_{22t}x_{12t})I_{1t}](N_2 - I_{2t} - R_{2t}) \\ &- \gamma_2 I_{2t}\} \\ &+\lambda_{1t}^R \gamma_1 I_{1t} + \lambda_{2t}^R \gamma_2 I_{2t} \end{aligned} \tag{7'}$$

The optimality conditions are:

$$\frac{\partial H}{\partial x_{iit}} = e^{-\rho t} N_i \left(\frac{\phi_0 N_i}{X_{it}} + \frac{\sigma}{x_{iit}} \right) X_{it} (x_{iit})^{\sigma}$$

$$+ \lambda_{it}^{I} \left(2\beta_0 x_{iit} I_{it} + \beta_2 x_{jit} I_{jt} \right) (N_i - I_{it} - R_{it})$$

$$+ \lambda_{jt}^{I} \beta_1 x_{jit} I_{it} \left(N_j - I_{jt} - R_{jt} \right) = 0, \quad i = 1, 2,$$

$$\frac{\partial H}{\partial x_{ijt}} = e^{-\rho t} \left[N_i \theta \left(\frac{\phi_2 N_i}{X_{jt}} + \frac{\sigma}{x_{ijt}} \right) X_{jt} \left(x_{ijt} \right)^{\sigma} + \phi N_j N_i \left(x_{jjt} \right)^{\sigma} \right] \tag{8'}$$

$$+ \lambda_{it}^{I} \beta_1 x_{jjt} I_{jt} (N_i - I_{it} - R_{it}) + \lambda_{jt}^{I} \beta_2 x_{jjt} I_{it} \left(N_j - I_{jt} - R_{jt} \right) = 0,$$

$$i = 1, 2, \quad i \neq j.$$

Also, for the co-state variables,

$$\frac{d\lambda_{it}^{I}}{dt} = -\frac{\partial H}{\partial I_{it}}$$

$$= -\lambda_{it}^{I} \left[\beta_0 (x_{iit})^2 (N_i - 2I_{it} - R_{it}) - \left(\beta_1 x_{ijt} x_{jjt} + \beta_2 x_{iit} x_{jit} \right) I_{jt} - \gamma_i \right] \tag{9'}$$

$$- \lambda_{jt}^{I} \left(\beta_1 x_{jit} x_{iit} + \beta_2 x_{jjt} x_{ijt} \right) \left(N_j - I_{jt} - R_{jt} \right) + \lambda_{it}^{R} \gamma_i + e^{-\rho t} c,$$

$$\frac{d\lambda_{it}^{R}}{dt} = -\frac{\partial H}{\partial R_{it}} = 0.$$

3.2 Comparing to the Static Cases

Now, consider static and/or decentralized cases to compare to the conditions derived in the previous subsection. First, characterize the nature of externality using a static model. Suppose, for a susceptible, the probability getting infected depends simply on his/her behavior:

$$\tilde{P}_i^S = \tilde{\eta}_0 x_{ii}^S + \tilde{\eta}_1 x_{ij}^S + \tilde{\eta}_2 x_{ii}^S, \tag{11}$$

A self-interested resident maximizes the expected utility:

$$EU_i^S = \left(1 - \tilde{P}_i^S\right)\left[\bar{X}_i\left(x_{ii}^S\right)^\sigma + \theta \bar{X}_j\left(x_{ij}^S\right)^\sigma\right] + \tilde{P}_i^S\left[\bar{X}_i\left(x_{ii}^S\right)^\sigma + \theta \bar{X}_j\left(x_{ij}^S\right)^\sigma - C\right] \tag{12}$$

The first-order conditions are:

$$\begin{aligned} \frac{\partial EU_i^S}{\partial x_{ii}^S} &= \frac{\sigma}{x_{ii}^S}\bar{X}_i\left(x_{ii}^S\right)^\sigma - (\tilde{\eta}_0 + \tilde{\eta}_2)C = 0, \\ \frac{\partial EU_i^S}{\partial x_{ij}^S} &= \frac{\sigma}{x_{ij}^S}\bar{X}_j\left(x_{ij}^S\right)^\sigma - \tilde{\eta}_1 C = 0, \quad i = 1, 2, \quad i \neq j. \end{aligned} \tag{13}$$

Next, consider the command-optimum of a static model of this type. Suppose the “actual” rate of infection, which takes into account the secondary or later infections, is represented by ηs instead of $\tilde{\eta}$s:

$$I_i^S = P_i^S S_i = \left(\eta_0 x_{ii}^S + \eta_1 x_{ij}^S + \eta_2 x_{ii}^S\right) S_i, \tag{14}$$

The plannner’s maximand is,

$$EV_i^S = \left(S_i - I_i^S\right)\left[X_i\left(x_{ii}^S\right)^\sigma + \theta X_j\left(x_{ij}^S\right)^\sigma\right] + I_i^S\left[X_i\left(x_{ii}^S\right)^\sigma + \theta X_j\left(x_{ij}^S\right)^\sigma - C\right] \tag{15}$$

The optimal conditions are:

$$\begin{aligned} \frac{\partial EV_i^S}{\partial x_{ii}^S} &= S_i\left(\frac{S_i}{X_i} + \frac{\sigma}{x_{ii}^S}\right) X_i\left(x_{ii}^S\right)^\sigma - S_i(\eta_0 + \eta_2)C = 0, \\ \frac{\partial EV_i^S}{\partial x_{ij}^S} &= S_i\theta\left(\frac{\phi S_i}{X_j} + \frac{\sigma}{x_{ij}^S}\right) X_j\left(x_{ij}^S\right)^\sigma + \phi S_j S_i\left(x_{jj}^S\right)^\sigma - S_i\eta_1 C = 0, \end{aligned} \tag{16}$$

$$i = 1, 2, \quad i \neq j.$$

The first term in the parentheses in each equation of (16) corresponds to agglomeration economies and, without taking account such as in (13), activity levels tend to be lower. The second term in the second equation as well as the last term in each equation, which is higher than that of (13) in per-capita terms, illustrate the externalities caused by the infectious disease. Without considering them, activities tend to exceed the optimal levels.

3.3 Comparing to the Dynamic Decentralized Case

Moreover, consider the case in which the model is dynamic but decentralized. Suppose that a resident decides the level of consumption irrespective of the externality he/she causes. That is, he/she takes Xs, Is and Ss as given. Once infected in period $t = \tau$, the probability of getting recovered is γ_i in every period. Therefore, the expected loss is:

$$\tilde{C}_{i\tau} = \int_{\tau}^{T} e^{-\rho(t-\tau)} e^{-\gamma_i(t-\tau)}\, c dt = \frac{c}{\gamma_i + \rho}\left(1 - e^{-(\gamma_i+\rho)(T-\tau)}\right) \tag{17}$$

Then, in each period τ, each self-interested resident maximizes:

$$\tilde{V}_{i\tau}^{Z} = \int_{\tau}^{T} e^{-\rho(t-\tau)} \left[\bar{X}_{it}\left(x_{iit}^{Z}\right)^{\sigma} + \theta\bar{X}_{jt}\left(x_{ijt}^{Z}\right)^{\sigma}\right]dt$$
$$-\left(\beta_0\bar{S}_{it}\bar{I}_{it}x_{iit}^{S}x_{iit}^{I} + \beta_1\bar{S}_{it}\bar{I}_{jt}x_{ijt}^{S}x_{jjt}^{I} + \beta_2\bar{S}_{it}\bar{I}_{jt}x_{iit}^{S}x_{jit}^{I}\right) \tag{18}$$
$$\frac{c}{\gamma_i + \rho}\left(1 - e^{-(\gamma_i+\rho)(T-\tau)}\right)$$

In the case of an uninfected resident (i.e., $Z = S$,) evaluating at τ and differentiating with respect to xs,

$$\frac{\partial\tilde{V}_{it}^{S}}{\partial x_{iit}^{S}} = e^{-\rho t}\frac{\sigma}{x_{iit}^{S}}\bar{X}_{it}\left(x_{iit}^{S}\right)^{\sigma} - \left(\beta_0\bar{S}_{it}\bar{I}_{it}x_{iit}^{I} + \beta_2\bar{S}_{it}\bar{I}_{jt}x_{jit}^{I}\right)$$
$$\frac{c}{\gamma_i + \rho}e^{-\rho t}\left(1 - e^{-(\gamma_i+\rho)(T-\tau)}\right) = 0, \quad i = 1, 2, \quad i \neq j. \tag{19}$$

$$\frac{\partial \tilde{V}_{it}^{S}}{\partial x_{ijt}^{S}} = e^{-\rho t}\frac{\sigma}{x_{ijt}^{S}}\bar{X}_{jt}\left(x_{ijt}^{S}\right)^{\sigma} - \beta_1 \bar{S}_{it}\bar{I}_{jt}x_{jjt}^{I}$$

$$\frac{c}{\gamma_i + \rho}e^{-\rho t}\left(1 - e^{-(\gamma_i+\rho)(T-\tau)}\right) = 0, \quad i = 1, 2, \quad i \neq j.$$

Comparing (19) to (8), one may find the differences that correspond to the ones found in the previous subsection. Also, one can easily show that the last term in each equation in (19) essentially correspond to λs in (8). In words, the market (self-interested) allocation tends to underestimate the role of activities when the agglomeration economies matter but overestimate them when secondary and inter-regional infections may occur. To correct for those externalities, the corresponding rate of charges or physical restrictions such as lock-down are necessary.

3.4 Externalities at Steady State: Simplest Case

In addition, this subsection illustrates the steady state with a simple numerical example. Consider, first, the following simplest SIS (discrete) setting in which $\beta_1 = \beta_2 = 0$:

$$\dot{S}_{it} = -\beta_{0iit}I_{it}S_{it} - \gamma_i I_{it}$$
$$\dot{I}_{it} = \beta_{0iit}I_{it}S_{it} - \gamma_i I_{it}, \tag{20}$$

Figure 1 depicts the changes in S and I when $\beta_0 = 0.3$, $x^S_{iit} = x^I_{iit} = 1$ (without control,) and $\gamma_i = 0.2$ in each region. By (20), the (non-zero) steady state in which $\dot{I}_{it} = 0$ is characterized by the following equation:

$$S_{it} = \frac{\gamma_i}{\beta_{0iit}}, \tag{21}$$

which equals $2/3$ in the case of Figure 1. When the number of newly infected equals the recovered, the steady state is achieved. When β_{0iit} can be controlled, for example, in the case of the "Uniform Control" in subsection 3.1,

$$I_{it} = N_i - \frac{\gamma_i}{\beta_0 (x^*_{ii})^2}. \tag{22}$$

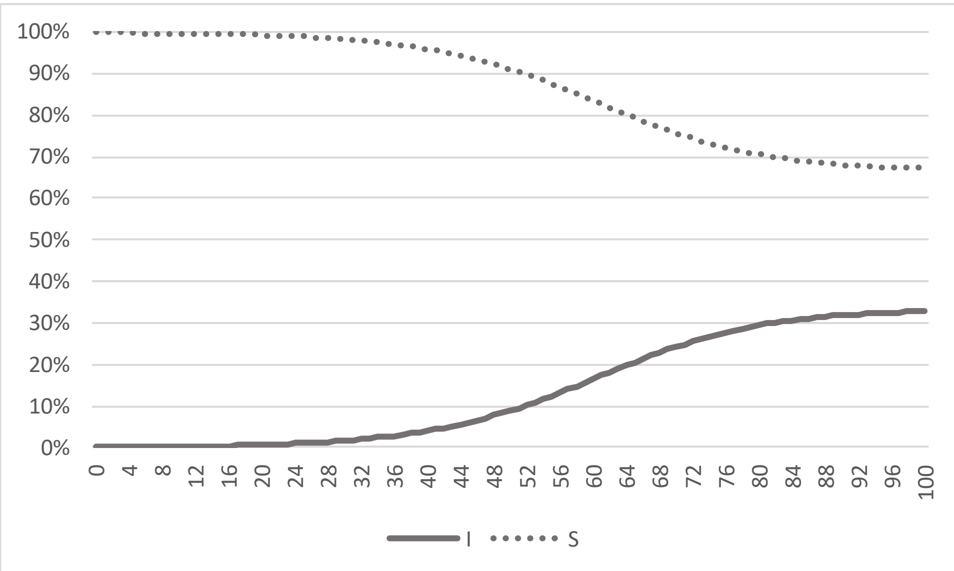

Figure 1: Simple SIS model without Control

Thus, around the vicinity of the steady state,

$$\frac{\Delta I_{it}}{\Delta x_{ii}} \cong \frac{2\gamma_i}{\beta_0 (x^*_{ii})^3} \Delta x_{ii}. \tag{23}$$

which corresponds to η_0 in the static case of subsection 3.2 and considerably large. Inverse of (22) is,

$$x_{ii}^{*} = \left(\frac{\gamma_i}{\beta_0}\frac{1}{N_i - I_{it}}\right)^{\frac{1}{2}}, \tag{24}$$

which correspond to $\dot{I} = 0$ curve in the phase diagram of Figure (2). In addition, in this particular case, for $T \to \infty$, (9) reduces to,

$$\frac{d\lambda_{it}^{I}}{dt} = -\frac{\partial H}{\partial I_{it}} = -\lambda_{it}^{I}[\beta_0(x_{iit})^2(N_i - 2I_{it}) - \gamma_i] + e^{-\rho t}c \tag{25}$$

Replacing λ_{it}^{I} with $\lambda_{it}^{I} = e^{-\rho t}\mu_{it}^{I}$,

$$\frac{d\mu_{it}^{I}}{dt} = -\mu_{it}^{I}[\beta_0(x_{iit})^2(N_i - 2I_{it} - R_{it}) - \gamma_i - \rho] + c \tag{26}$$

The curve for $\dot{\mu} = 0$ is drawn such as in Figure 2.

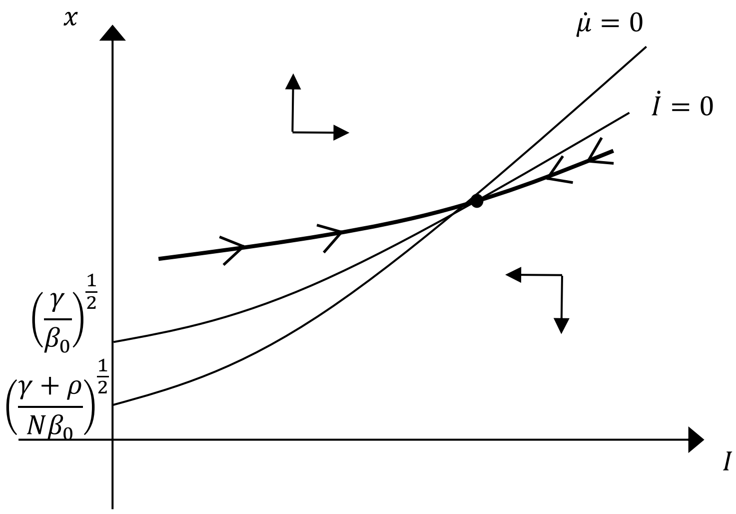

Figure 2: An Example of the Phase Diagram around the Steady State

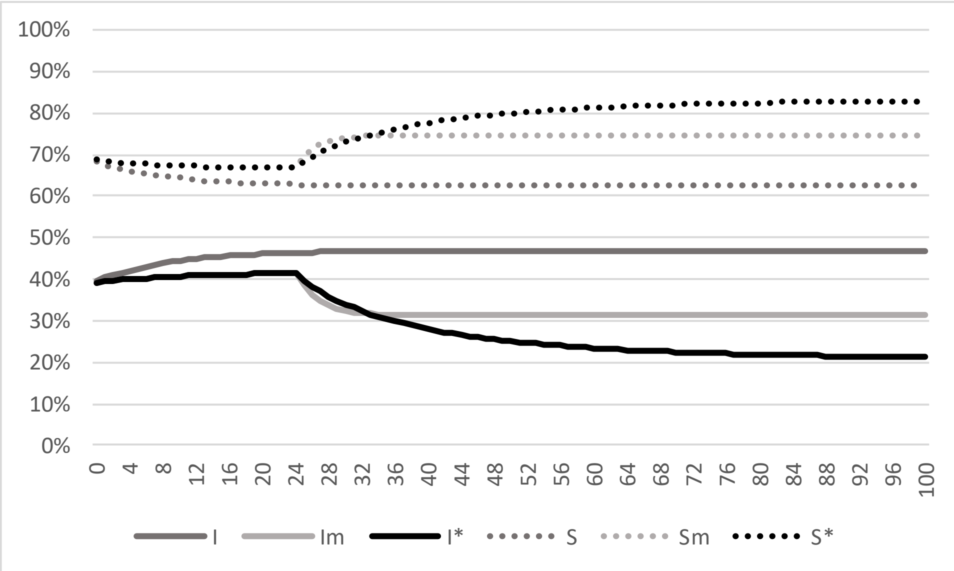

Figure 3: No control, market and optimal paths

Illustrate the change in the steady state when the economy is already in the situation characterized by (21) or in Figure 1. Consider, for simplicity, the cases in which $T \to \infty$, $\Phi(I_{1T}, I_{2T}) \to \infty$, and $\phi_0 = \phi_2 = 0$. In the decentralized economy without travels ($\theta = \beta_1 = \beta_2 = 0$,) the infected residents do not impose restrictions on themselves (x_{iit}^I=1) and the condition in (19) reduces to:

$$\sigma\phi_0\left(x_{iit}^S\right)^{\sigma-1} - \beta_0 S_{it} I_{it} x_{iit}^I \frac{c}{\gamma_i + \rho} = 0. \tag{19'}$$

Also, the optimal control at steady state maximizes the following with respect to (22):

$$\begin{aligned} V_i &= \int_0^\infty e^{-\rho t}\left\{\phi_0 N_i (x_{ii}^*)^\sigma - c\left(N_i - \frac{\gamma_i}{\beta_0 (x_{ii}^*)^2}\right)\right\} dt \\ &= \frac{1}{\rho}\left\{\phi_0 N_i (x_{ii}^*)^\sigma - c\left(N_i - \frac{\gamma_i}{\beta_0 (x_{ii}^*)^2}\right)\right\}. \end{aligned} \tag{6''}$$

The curves in Figure 3 indicate the paths without control, decentralized control by (19') and optimal control by (6''), respectively, for $\sigma = 0.5$, $\phi_0 = 1$, $c = 1.5$, $N_i = 7$, and $\rho \to 0$.

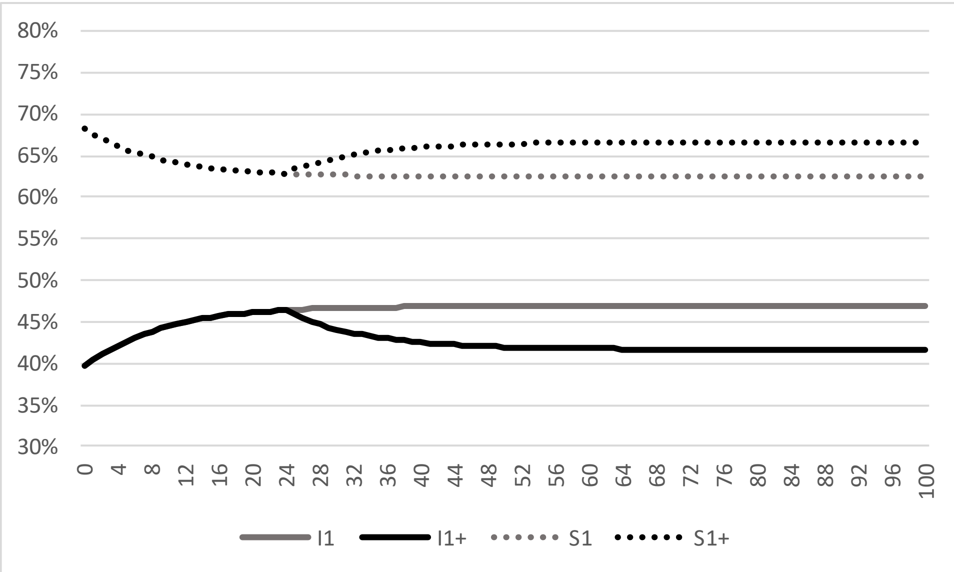

Figure 4: With Travels (no control and controlled paths)

With travels, taking into account the symmetry, the steady state (21) changes to:

$$S_{it} = \frac{\gamma_i}{\beta_0(x_{iit})^2 + \beta_1 x_{ijt}x_{jjt} + \beta_2 x_{iit}x_{jit}}, \tag{27}$$

The planner need to control both x_{ii}s and x_{ij}s so that to obtain an explicit solution for the condition (6') is somewhat difficult. Thus, this subsection considers the case in which only x_{ij}s, the travels, are controlled. Then, (6') is rewritten as:

$$V = \int_0^\infty e^{-\rho t}\{\phi_0 N_1[1 + \theta(x_{12}^*)^\sigma] - cI_1^* + \phi_0 N_2[1 + \theta(x_{21}^*)^\sigma] - cI_2^*\}dt \tag{28}$$

$$= \frac{1}{\rho}\{\phi_0 N_1[1+\theta(x_{12}^*)^\sigma] - cI_1^* + \phi_0 N_2[1+\theta(x_{21}^*)^\sigma] - cI_2^*\}$$

Moreover, the decentralized behavior is characterized by (19). Figure 4 presents the difference between the path controlled by (28) and the one without a control (the decentralized -path coincides with the latter as $x_{ij} = 1$ is the solution for the above parameter values.

3.5 Externalities at a Particular Point: SIR Case

When SIR is assumed, for $\beta_{0iit} = 0.3$ and $\gamma_i = 0.2$, the changes in S, I and R are depicted such as the light-colored curves in Figure 5. Note that there is no longer a steady state because recovered people are not to be infected again. Instead, there is a local maximum of I. Again, (22) and (23) hold around the vicinity of the corresponding point. Deriving the optimal paths is extremely difficult because no "steady state" in this case and the optimality conditions in (8')-(9') must be fully used.

Because $\dot{I}_{it} = 0$ when I_{it} hits its maximum, this subsection considers the second-best rule in which (27)-(28) are still applied for control. Then if the control starts when I_{1t} hits its maximum, the paths change to the darker ones in Figure5. Also, if the same rule is applied form the beginning, the situation in Figure 6 is to occur and almost no infection is to be seen in the second region.

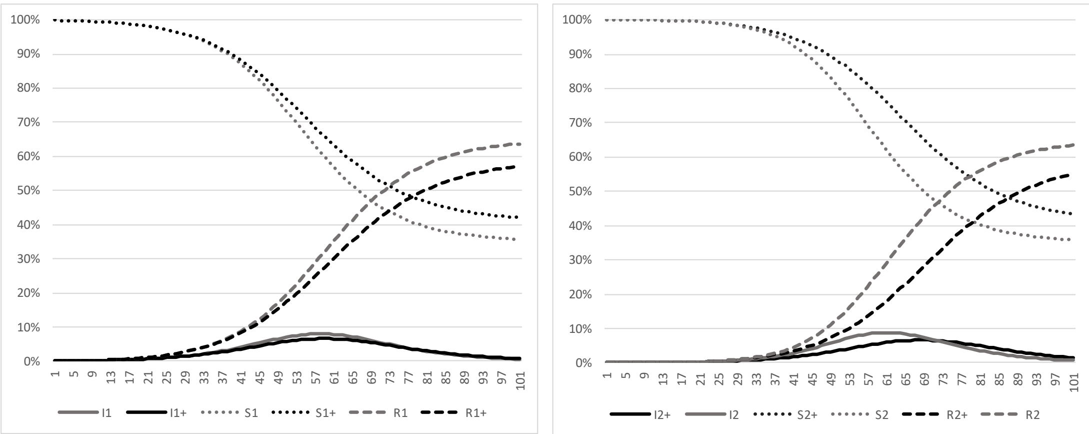

Figure 5: SIR Case

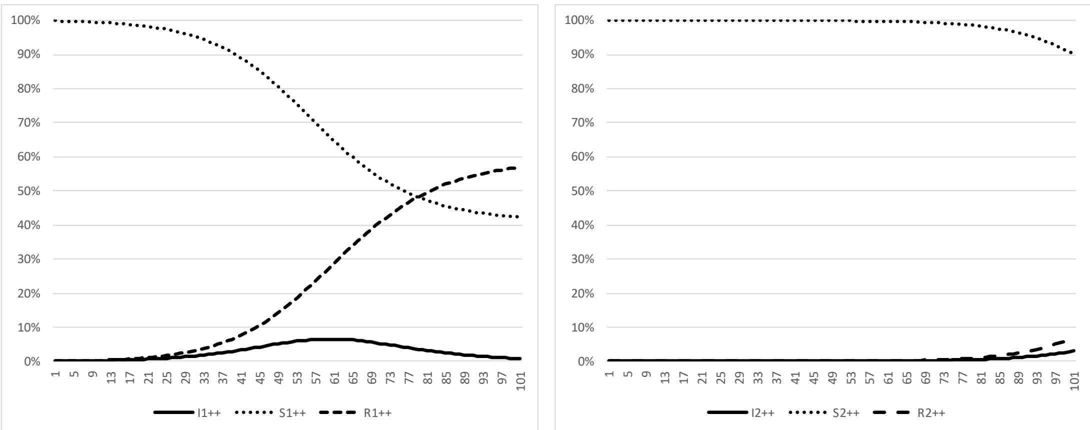

Figure 6: SIR Case (applied from the beginning)

4. Conclusion

This paper has modelled the optimal and market level of consumption in an economy with two regions when infectious disease may spread. Typical SIS and SIR dynamic model is extended to characterize the two-region economy. The corresponding dynamic optimal control problem has been set up and the Hamiltonian is presented. Self-interested behavior ignores agglomeration economy as well as the negative externality of the disease such as the secondary and later infections and results in under- or over-consumption (activity) levels. In general, corresponding corrections such as charges or regulations are required. Some simple examples are presented to illustrate the steady state and local extremum of SIS and SIR models, respectively.

The possible extensions of this paper includes 1) a detailed simulation with optimal control of xs, specifying the paths of λs and using realistic estimates of the parameters, 2) considering a case in which the interaction between two regions appears in more prominent way (e.g., two peaks,) and 3) identifying the optimal levels of regulations.

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