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Utility of Change and Lifetime Consumption

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Abstract

This study investigates the behavior and welfare of an individual whose instantaneous utility is a function of the change in his/her consumption, as well as the absolute level.

The behavior is largely affected by the initial and terminal conditions on the levels of consumption. Without a terminal condition, an individual, who has a preference for positive change, tends to postpone his/her consumption until later. With a condition, the consumption path is largely determined by backward (and forward) "consumption smoothing."

Several applied topics such as Kahneman and Tversky-type modeling and capital accumulation are also discussed.

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1. Introduction

Anyone who is acquainted with economic theory knows that a model is some simplification of the real world. Most of the fundamental models, however, have been considered as good approximations of it: In spite of some anomalies, major mechanisms seem to be properly taken into account and deduced results would not be far from the reality, ceteris paribus. Among them is Discounted Utility Model, which has been used in most studies of intertemporal consumption choices since it was introduced by Samuelson (1937).

Yet, if one is serious enough, he/she may find that there is little intuitive or empirical support for the Discounted Utility Model. The model assumes lifetime satisfaction or happiness of an individual is described by the (discounted) sum of his/her utilities over time, each of which is a function of income or consumption in the corresponding period. Then, it is possible that some consumption profile which enables an individual to have “miserable days in the first half of life but happy ones in the latter half” is equivalent to another profile which brings “happy days in the first half of life but miserable ones in the latter half” (with some adjustments on the consumption levels taking into account the discount factor.) Not to mention Aesop’s *the Ant and the Grasshopper,* few people may agree that they are equivalent.

Actually, since the seminal work of Easterlin (1974, updated in 1995), numerous studies have discussed whether or not a large part of human satisfaction is characterized by the absolute level of income (or consumption.) As Clark et al. (2008) summarizes, many studies have admitted the existence of "Easterlin Paradox," which presumes limited or little contribution of absolute levels to satisfaction.[1] [2]

The most straightforward interpretation of "Easterlin Paradox" is that one's satisfaction is based on some comparison with others or one's own state in the past. The idea dates back to the "Relative Income Hypothesis" of Duesenberry (1949). Taking into account the hypothesis, empirical findings such as Di Tella et al. (2003), Stutzer (2004), Di Tella et al. (2005) have reinforced the interpretation.

Some behavioral economists have also focused on the issue. Kahneman and Thaler (1991) states "comparisons to others and especially to one's past determine the standard of satisfaction with income." The models of habit formation, such as Pollak (1970), have been used to characterize the human behavior based on a similar idea.

Extensions include those of Gilboa (1989), which explicitly considers utility variation between two consecutive periods, Loewenstein and Prelec (1993), which models global properties of a sequence and argues "Preference for Spreading" and

[1] Some studies, such as Stephenson and Wolfers (2008), have presented skepticism on "Easterlin Paradox." Frank (2012) further refutes such skepticism.

[2] Sasaki (2008) summarizes the evidences in Japan.

Shalev (1997), which takes into account the notion of loss-aversion and the prospect theory of Kahneman and Tversky (1979).

In the context of macroeconomic decision-making, Bordley (1986) and Frank (1989) consider continuous-time models. The latter, Frank (1989), and Frank and Hutchens (1993) mention the quality of life in the context of habit formation. Carroll et al. (1997) and Carroll et al. (2000) analyze saving behavior and capital growth by explicitly modeling the speed of adaptation.

Experimental studies on habit formation include Loewenstein (1987) and Lowenstein and Prelec (1991), which are about famous comparison among sequential combinations of “Eat at Home ,” “Fancy French” and “Fancy Lobster,” and Hsee and Abelson (1991) and Hsee et al. (1991), which are about hypothetical changes in salary (or academic performance.)

Recently, based on the experiment of Loewenstein (1987), Wakai (2008) and Wakai (2013) develop theoretical models which characterize “utility smoothing.”

In spite of those various contributions, there are few articles which model the lifetime behavior of an individual and argue his/her quality of life consistently in order to indicate the welfare implications of habit formation.

This study addresses the following research questions: 1) What is the (life-time) behavior of an individual who values the first-order change in consumption like? 2)

How significant are the initial and terminal conditions in such a (habit-formation) model? 3) How do the results change if the utility function (of change) is non-concave? 4) What if capital accumulation is explicitly considered?

On one hand, the approach of this study is somewhat close to Carroll et al. (1997) and Carroll et al. (2000), which actually derive several consumption paths using models of habit formation (although little argument on well-being is seen in their studies.[3]) On the other hand, the motivation behind this study is close to those of Frank (1989) and Frank and Hutchens (1993), which mention the quality of life of an individual (while they do not present the implications of their models completely.) This study lies between those two groups of studies and develops them.

The rest of the paper is organized as follows: Section 2 describes the general setting of the study. Section 3 and 4 consider the cases in which only the absolute level matters and the first-order change does, respectively. Section 5 considers the case where both factors are taken into account. Section 6 investigates the case in which utility function is not concave but of Kahneman Tversky-type . Section 7 considers capital accumulation. Section 8 concludes.

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[3] Also, their models, which seem to be arranged in order to obtain explicit solutions, are slightly different from that of this study.

2. Maximization Problem

Suppose the instantaneous utility of an individual is a function of the level of his/her consumption and its first-order change.[4] Denote the level of his/her consumption in period *t* by $c_t^{(0)}$ and the difference from the previous period by $c_t^{(1)} \equiv c_t^{(0)} - c_{t-1}^{(0)}$.[5] Then, consider the following simple additively separable function:

$$\tilde{u}_t = \alpha_0 \left(c_t^{(0)}\right)^{\beta_0} + \alpha_1 \left(c_t^{(1)} + \tilde{c}^{(1)}\right)^{\beta_1}, \tag{1}$$

where $\tilde{c}^{(1)} > 0$.

In the second term, $\tilde{c}^{(1)}$ is added to $c_t^{(1)}$ in order to allow zero or negative $c_t^{(1)}$, which is regularly observed in the real world. Assume that an individual is endowed with initial asset a_0 and lives for $0 \leq t \leq T$. Denoting the level of his/her asset remaining in period *t* by a_t, the intertemporal utility maximization problem is written as:[6] [7]

$$\max \sum_{t=1}^{T} \left[\left(\frac{1}{1+\rho} \right)^{t-1} \left\{ \alpha_0 \left(c_t^{(0)}\right)^{\beta_0} + \alpha_1 \left(c_t^{(1)} + \tilde{c}^{(1)}\right)^{\beta_1} \right\} \right], \tag{2}$$

subject to $a_t = (1+r)\left(a_{t-1} - c_t^{(0)}\right)$,

$c_0^{(0)} = \bar{c}_0^{(0)}, \quad c_T^{(0)} = \bar{c}_T^{(0)}, \quad a_0 = \bar{a}_0$ and

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[4] Yonemoto (2013) investigates more general settings.

[5] Note that there are several ways to define the "difference" around period *t* such as a weighted average of the differences around *t* ($c_t^{(1)} \equiv \alpha\left(c_t^{(0)} - c_{t-1}^{(0)}\right) + (1-\alpha)\left(c_{t+1}^{(0)} - c_t^{(0)}\right)$.) (1) simply uses the difference of the current value from the previous one. It can be regarded as a specific kind of habit formation model.

[6] One may assume utility is also a function of a. Refer to Yonemoto (2013) for example.

[7] Any realistic setting may have a non-negative constraint on $c_t^{(0)}$.

$$c_t^{(1)} \equiv c_t^{(0)} - c_{t-1}^{(0)},$$

where r and ρ are the interest rate and discount rate, respectively.[8]

One may wonder if the initial and terminal conditions are in fact necessary. Yet, it is natural to think that the first *change* in consumption in the life of an individual is the difference between the pre-birth (or childhood) level, which is determined biologically or by the intent of parents, and the level that he/she chooses arbitrarily for the first time. Similarly, one can imagine that the last change in his/her life is the difference between his/her last choice and the "afterlife" level in his/her conception.[9]

The structural equations are combined into a single equation by adding up:

$$a_0 = \sum_{t=1}^{T} \left\{ \left(\frac{1}{1+r} \right)^{t-1} c_t^{(0)} \right\} + \left(\frac{1}{1+r} \right)^{T} a_T . \tag{3}$$

The Lagrangian for (2) is, taking into account (3),

$$\begin{aligned} L = & \sum_{t=1}^{T} \left[\left(\frac{1}{1+\rho} \right)^{t-1} \left\{ \alpha_0 \left(c_t^{(0)} \right)^{\beta_0} + \alpha_1 \left(c_t^{(1)} + \tilde{c}^{(1)} \right)^{\beta_1} \right\} \right] \\ & + \sum_{t=1}^{T} \lambda_t \left\{ \left(c_t^{(0)} - c_{t-1}^{(0)} \right) - c_t^{(1)} \right\} \\ & + \mu \left\{ a_0 - \sum_{t=1}^{T} \left\{ \left(\frac{1}{1+r} \right)^{t-1} c_t^{(0)} \right\} - \left(\frac{1}{1+r} \right)^{T} a_T \right\}. \end{aligned} \tag{4}$$

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[8] Some studies of behavioral economics argue if the subjective discount rate changes as time passes by (e.g. Wolf (1970).) This study uses the simplest setting in which the rate is constant. In other words, the study considers the life-time consumption plan at the beginning.

[9] One who does not believe in the "afterlife" may presume the level to be zero.

Differentiating (4) with respect to the control variables $c_1^{(0)},\cdots,c_{T-1}^{(0)}$ and $c_1^{(1)},\cdots,c_T^{(1)}$, the following first-order conditions are derived:

$$\frac{\partial L^*}{\partial c_t^{(0)}}=\left(\frac{1}{1+\rho}\right)^{t-1}\alpha_0\beta_0\left(c_t^{(0)}\right)^{\beta_0-1}+\lambda_t-\lambda_{t+1}-\mu\left(\frac{1}{1+r}\right)^{t-1}=0\ ,$$

$$\text{for}\quad t=1,\cdots,T-1, \qquad (5)$$

$$\frac{\partial L^*}{\partial c_t^{(1)}}=\left(\frac{1}{1+\rho}\right)^{t-1}\alpha_1\beta_1\left(c_t^{(1)}+\tilde{c}^{(1)}\right)^{\beta_1-1}-\lambda_t=0, \qquad \text{for}\quad t=1,\cdots,T\ , \qquad (6)$$

Substituting λ_t and λ_{t+1} of (6) into (5), the following expression is obtained:

$$\left(\frac{1}{1+\rho}\right)^{t-1}\alpha_0\beta_0\left(c_t^{(0)}\right)^{\beta_0-1}+\left(\frac{1}{1+\rho}\right)^{t-1}\alpha_1\beta_1\left(c_t^{(1)}+\tilde{c}^{(1)}\right)^{\beta_1-1}$$

$$-\left(\frac{1}{1+\rho}\right)^{t}\alpha_1\beta_1\left(c_{t+1}^{(1)}+\tilde{c}^{(1)}\right)^{\beta_1-1}-\mu\left(\frac{1}{1+r}\right)^{t-1}=0 \quad \text{for}\quad t=1,\cdots,T-1. \qquad (7)$$

The first term in the left-hand side of (7) is associated with $c_t^{(0)}$, the level of consumption, while the second and third terms depend on $c_t^{(1)}$ and $c_{t+1}^{(1)}$, the changes. The following two sections investigate two extreme cases to interpret this condition clearly.

3. No "Utility of Change"

First, consider one extreme: The utility depends solely upon the level of consumption ($\alpha_1 = 0$.) Then, the problem reduces to that of a simple intertemporal choice with CRRA (constant relative risk aversion) utility. Condition (7) is rewritten to be:

$$\left(\frac{1}{1+\rho}\right)^{t-1} \alpha_0 \beta_0 \left(c_t^{(0)}\right)^{\beta_0 - 1} - \mu \left(\frac{1}{1+r}\right)^{t-1} = 0 . \qquad (8)$$

Thus,

$$c_{t+1}^{(0)} \begin{array}{c} > \\ = \\ < \end{array} c_t^{(0)} \quad \text{as} \quad r \begin{array}{c} > \\ = \\ < \end{array} \rho . \qquad (9)$$

Since the instantaneous utility is concave, an individual tends to avoid a fluctuation in consumption. With $r = \rho = 0$, the optimal consumption is constant over time. With positive r and ρ, the consumption path is skewed according to their relative sizes as Figure 3.1 illustrates.

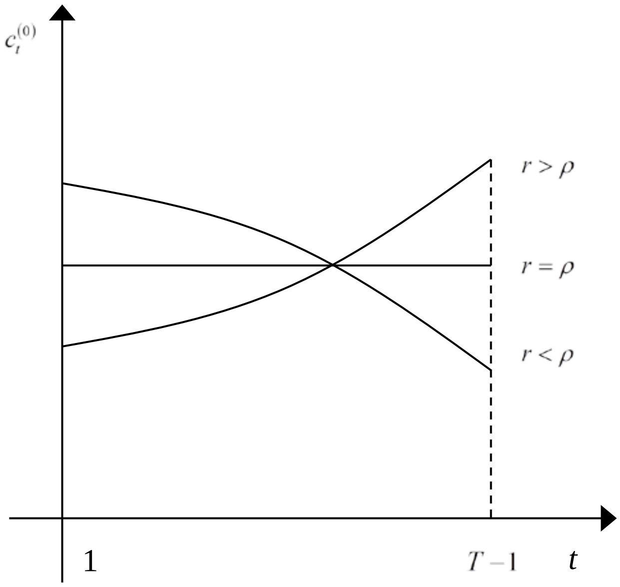

Figure 3.1 Optimal Consumption with No "Utility of Change"

4. Only "Utility of Change"

Next, consider the other extreme: The utility depends only on the changes in consumption ($\alpha_0 = 0$.) Then, condition (7) becomes:

$$\left(\frac{1}{1+\rho}\right)^{t-1} \alpha_1\beta_1\left(c_t^{(1)} + \tilde{c}^{(1)}\right)^{\beta_1 - 1} - \left(\frac{1}{1+\rho}\right)^{t} \alpha_1\beta_1\left(c_{t+1}^{(1)} + \tilde{c}^{(1)}\right)^{\beta_1 - 1} - \mu\left(\frac{1}{1+r}\right)^{t-1} = 0 . \tag{10}$$

The second term is negative so that $c_t^{(1)}$ needs to be balanced with $c_{t+1}^{(1)}$. If r and ρ are zero and a_0, the initial endowment, is large enough to make μ zero, $c_t^{(1)} = c_{t+1}^{(1)}$ (see Figure 4.1.) That is, an individual seeks constant change in consumption from the initial level $\bar{c}_0^{(0)}$ to the final level $\bar{c}_T^{(0)}$. (If the initial endowment is larger than the amount necessary to sustain the constant change, the remainder is abandoned.) Note that, if $\bar{c}_0^{(0)} > \bar{c}_T^{(0)}$, consumption decreases constantly as long as $c_t^{(1)} + \tilde{c}^{(1)} > 0$ is satisfied for all t.

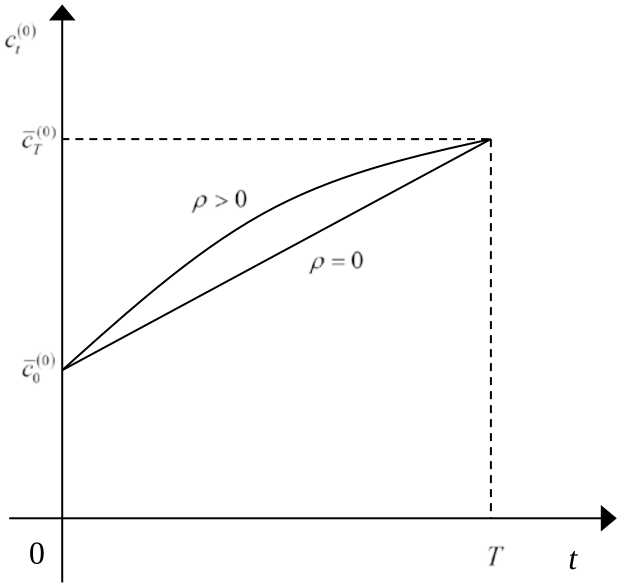

Figure 4.1 Optimal Consumption with Only "Utility of Change" ($\mu = 0$)

Figure 4.1 also illustrates the case in which $\rho > 0$ while $\mu = 0$ (and $r = 0$.) In this case, the increase in an earlier period is larger than the one in a later period because of the positive discount rate. Figure 4.2 illustrates other cases assuming $\rho = 0$. $\mu > 0$ occurs when the initial endowment is not enough to make the changes depicted in Figure 4.1 possible. To reduce the area under the curve, which is the life-time consumption, the curve is bowed downward. As a result, the amount of the change increases as time passes by. If r increases when $\mu > 0$, the income effect raises the curve upward. As a result, it reduces convexity.

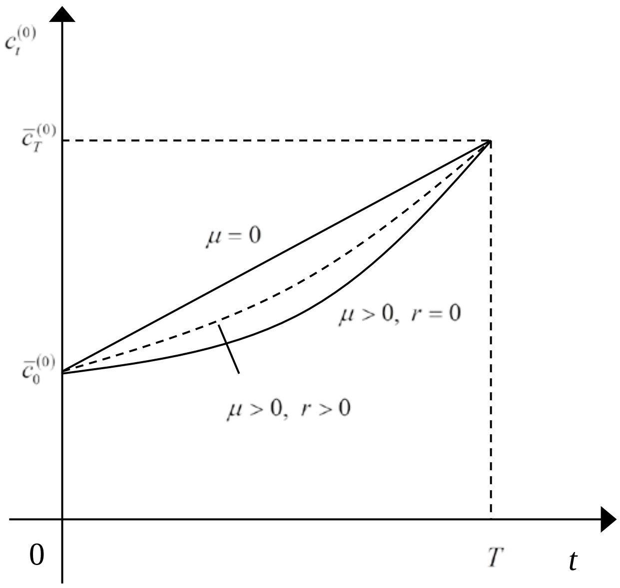

Figure 4.2 Various Cases When $\rho = 0$

Variable Initial and Terminal Conditions

Now, consider the cases in which the initial and/or terminal conditions are relaxed. 1) If both are relaxed ($c_0^{(0)}$ and $c_T^{(0)}$ can be controlled by the individual,) the solutions are indeterminate. In this case, by reducing $c_0^{(0)}$, one can raise his/her life-time utility as much as he/she can. In other words, since only the change matters, making his/her childhood as "miserable" as possible, one can be better off. 2) Similarly, if $c_T^{(0)}$ is fixed but $c_0^{(0)}$ is variable, one can increase his/her life-time utility by choosing the smallest possible $c_0^{(0)}$. Again, the solutions are indeterminate. 3) However, if $c_T^{(0)}$ can be chosen while $c_0^{(0)}$ is fixed, things are different. In addition to (5), the following condition is necessary to characterize the optimal $c_T^{(0)}$:

$$\frac{\partial L^*}{\partial c_T^{(0)}} = \left(\frac{1}{1+\rho}\right)^{T-1} \alpha_0 \beta_0 \left(c_T^{(0)}\right)^{\beta_0 - 1} + \lambda_T - \mu \left(\frac{1}{1+r}\right)^{T-1} = 0 \ , \tag{11}$$

where $\alpha_0 = 0$ in the case here.

Although one can increase the lifetime utility by raising $c_T^{(0)}$, now he/she needs to take into account the (life-time) income constraint. That is, multiplier μ is positive when $c_T^{(0)}$ is optimally chosen. As a result, the corresponding path of $c_t^{(0)}$ can be drawn such as those of $\mu > 0$ in Figure 4.2. Figure 4.3 illustrates the optimal path derived numerically for $T = 7$, $r = \rho = 0$, $\alpha_1 = 0.5$, $\beta_1 = 0.5$, $\bar{c}_0^{(0)} = 0$, $\tilde{c}^{(1)} = 10$ and $a_0^{(0)} = 1000$.[10] (μ is calculated to be 0.011.)

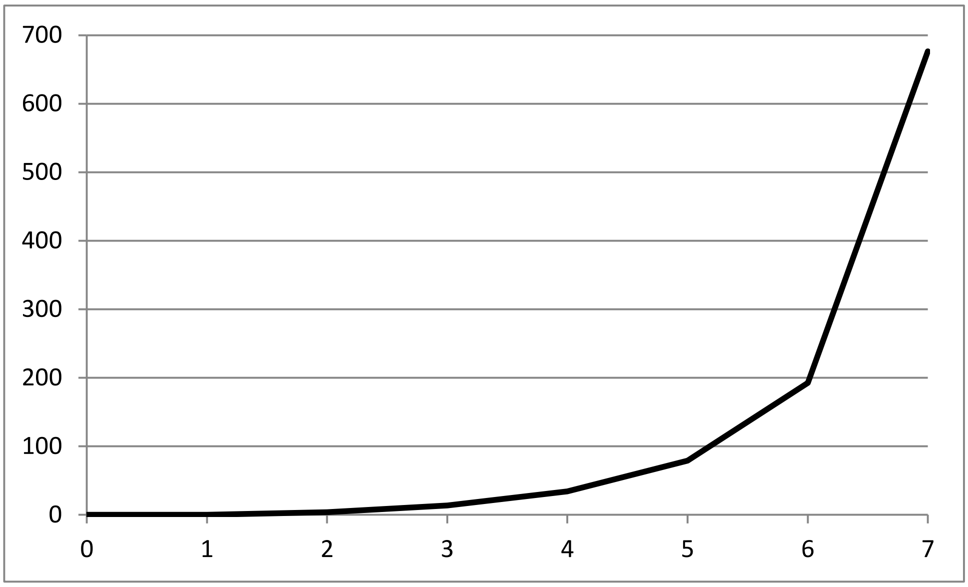

Figure 4.3 A Numerical Example

[10] The elasticity of intertemporal substitution $1/(1-\beta_0)$ is often estimated to be less than 1 so that β_0 (and α_0) can be negative. However, since this study also deals with α_1 and β_1, the parameters on the changes, the estimates of other studies cannot be applied directly. Moreover, assuming negative β s (and α s) brings about a serious problem when a Kahneman and Tversky-type function is defined in Section 6.Thus, this study simply uses positive values for numerical calculations.

5. Both Factors Taken Into Account

In this section, consider the case in which both the level of consumption and its change constitute the utility: $\alpha_0, \alpha_1 > 0$. At first, assume fixed $\bar{c}_0^{(0)}$ and $\bar{c}_T^{(0)}$ again. Noting that the entire part of equation (7) matters now, for $r = \rho = 0$,

$$c_{t+1}^{(1)} \begin{array}{c} > \\ = \\ < \end{array} c_t^{(1)} \quad \text{as} \quad c_t^{(0)} \begin{array}{c} > \\ = \\ < \end{array} \left(\frac{\alpha_0 \beta_0}{\mu} \right)^{\frac{1}{1-\beta_0}}.$$

That is, if the level of consumption is larger (smaller) than some threshold, the curve must be convex (concave.) Figure 5.1 summarizes possible patterns for $\bar{c}_0^{(0)} < \bar{c}_T^{(0)}$. The optimal path essentially inherits the properties of the curves in Section 4 as well as the ones in Section 3. The curve is drawn so that some (absolute) level of

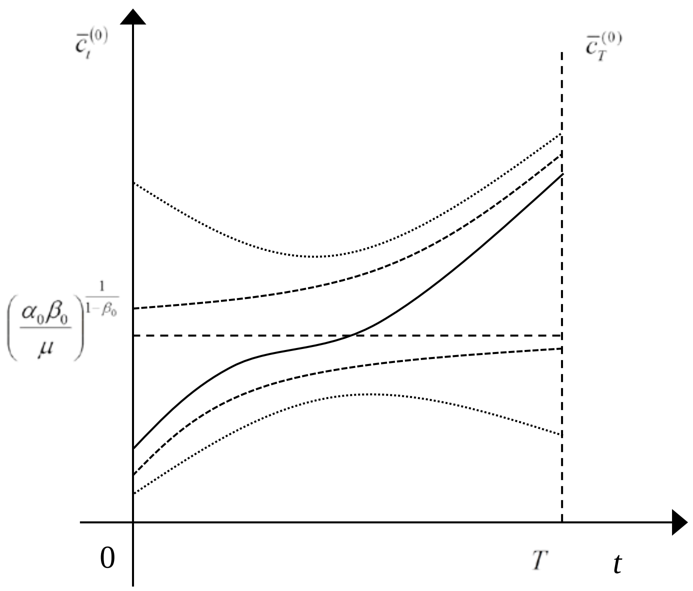

Figure 5.1 Optimal Paths for $\alpha_0, \alpha_1 > 0$

consumption is kept as long as possible. At the same time, it smoothly connects the start point with the end point to reduce the first-order fluctuation.

Variable Initial and Terminal Conditions

Next, consider the cases where the initial and/or terminal levels of consumption can be controlled. As for the initial level, since it is not accounted for as a part of lifetime utility (by definition,) reducing it simply has a positive effect on the change in the first period; Like in the case of Section 4, the solution is indefinite.

As for the terminal level of consumption, raising it has positive effects on both the level itself as well as the first-order change. Thus, if it can be controlled, (life-time) income needs to be exhausted ($\mu > 0$.) The curve typically has convex as well

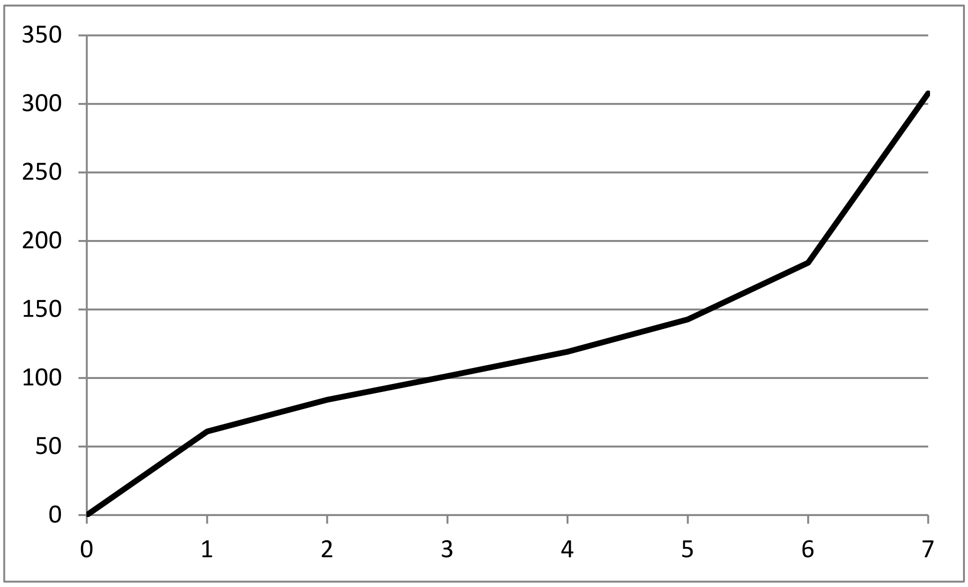

Figure 5.2 Numerical Solution of an Example

as concave parts such as the solid curve in Figure 5.1. Figure 5.2 shows the numerical solution when $T = 7$, $r = \rho = 0$, $\alpha_0 = 1$, $\alpha_1 = 0.5$, $\beta_0 = \beta_1 = 0.5$, $\tilde{c}^{(1)} = 10$, $c_0^{(0)} = 0$ and $a_0^{(0)} = 1000$. (μ is calculated to be 0.05.)

Comparing the Cases With and Without the Utility of Change

Finally, investigate what happens to the consumption path when α_1 increases from zero (i.e. the change in consumption has no effect on utility) to some positive number (i.e. the change has some effect,) keeping the level of initial asset constant. Suppose $c_0^{(0)}$ is fixed and $c_T^{(0)}$ is variable. Also, assume $r = \rho = 0$ for simplicity. Then, $\alpha_1 = 0$ yields $c_t^{(0)} = \overline{a}_0^{(0)} / T$ for all *t* so that the optimal path looks like the straight line in Figure 3.1.[11] If $c_0^{(0)}$ is also equal to $\overline{a}_0^{(0)} / T$, an increase in α_1 alters the optimal consumption path such as in Figure 5.3.[12] The Proof is given as follows:

i) To show $c_t^{(0)}$ has at most one local minimum

First, show that if $c_{\hat{t}}^{(1)} < 0$ and $c_{\hat{t}+1}^{(1)} \geq 0$ for some $\hat{t}$, then $c_t^{(1)} < 0$ for all $t \leq \hat{t}$.

By (7),

[11] Note that $c_T^{(0)}$ is fixed in Section 3 and thus $c_t^{(0)} = \left(a_0^{(0)} - \overline{c}_T^{(0)}\right)/(T-1)$.

[12] Without assuming $c_0^{(0)}$ is fixed (equal to $\overline{a}_0^{(0)} / T$) and $c_T^{(0)}$ is variable, comparison makes little sense: If $c_0^{(0)}$ is variable, as has been argued so far, the solution is indefinite. If $c_0^{(0)}$ is fixed but not equal to $\overline{a}_0^{(0)} / T$, its value irregularly affects the levels of the consumption in the first several periods. Moreover, if $c_T^{(0)}$ is fixed and smaller than $\overline{a}_0^{(0)} / T$, some part of the asset can be abandoned. Even if it is larger than or equal to $\overline{a}_0^{(0)} / T$, its value mostly determines the levels of the consumption in the last several periods.

$$\mu = \alpha_0\beta_0\left(c_{\hat{t}}^{(0)}\right)^{\beta_0-1} + \alpha_1\beta_1\left(c_{\hat{t}}^{(1)} + \tilde{c}^{(1)}\right)^{\beta_1-1} - \alpha_1\beta_1\left(c_{\hat{t}+1}^{(1)} + \tilde{c}^{(1)}\right)^{\beta_1-1}$$

$$= \alpha_0\beta_0\left(c_{\hat{t}-1}^{(0)}\right)^{\beta_0-1} + \alpha_1\beta_1\left(c_{\hat{t}-1}^{(1)} + \tilde{c}^{(1)}\right)^{\beta_1-1} - \alpha_1\beta_1\left(c_{\hat{t}}^{(1)} + \tilde{c}^{(1)}\right)^{\beta_1-1}.$$

$$\text{for} \quad \hat{t} = 2,\cdots,T-1, \qquad (12)$$

Noting $c_{\hat{t}}^{(1)} = c_{\hat{t}}^{(0)} - c_{\hat{t}-1}^{(0)} < 0$, it follows that $c_{\hat{t}-1}^{(1)} < c_{\hat{t}}^{(1)}$. For the case $\hat{t} = T$ ($c_T^{(1)} < 0$,) by (11) and (12), $c_{T-1}^{(1)} < c_T^{(1)} < 0$. In either case, by applying (12) repeatedly, $c_1^{(1)} < c_2^{(1)} < \cdots < c_{\hat{t}-1}^{(1)} < c_{\hat{t}}^{(1)} < 0$. Therefore, $c_0^{(0)} > c_1^{(0)} > c_2^{(0)} > \cdots > c_{\hat{t}-1}^{(0)} > c_{\hat{t}}^{(0)}$. That is, $c_t^{(0)}$ has at most one local minimum.

ii) Illustrating the path

i) implies that $c_t^{(0)}$ is monotonically increasing, decreasing or has a local minimum. However, if $c_t^{(0)}$ monotonically increasing from $c_0^{(0)} = \overline{a}_0^{(0)}/T$, the asset is

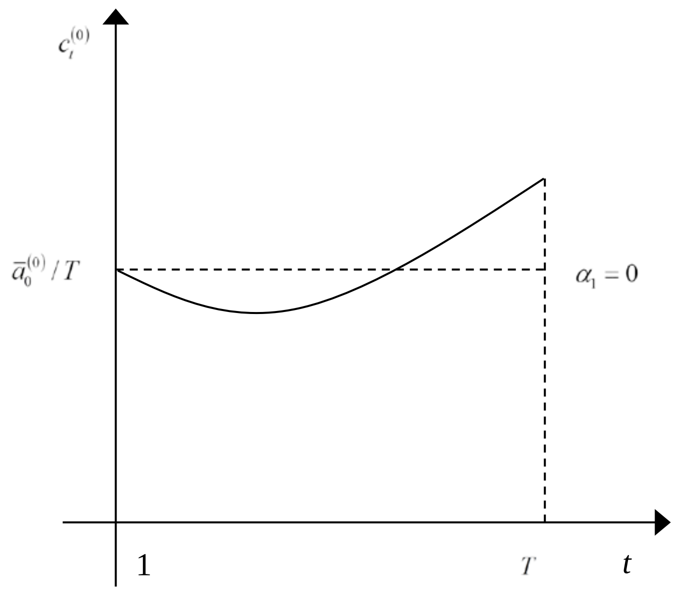

Figure 5.3 Comparing the Cases With and Without the Utility of Change

used up before T. Moreover, if it is monotonically decreasing or has a local minimum but is ending with $c_T^{(0)} \leq \overline{a}_0^{(0)} / T$, $c_t^{(0)} = \overline{a}_0^{(0)} / T$ for all t is obviously better. Thus, $c_t^{(0)}$ has a local minimum and $c_T^{(0)} > \overline{a}_0^{(0)} / T$.

6. Kahneman and Tversky-Type Function

So far, the (instantaneous) utility function has been well-behaved so that the optimal paths have not had irregular jumps. However, if the utility function has any convex parts, a large jump can be observed.

As illustrated in Figure 6.1, the value function proposed by Kahneman and Tversky (1979) is convex for “losses.” Instead of defining the “gains” or “losses” by using the level of consumption and some arbitrarily-chosen reference point, this study has characterized them by the (first-order) differences in consumption.[13] The model can be extended by assuming the followings:

$$\frac{\partial u}{\partial c_t^{(1)}} > 0,$$

$$\frac{\partial^2 u}{\partial \left(c_t^{(1)}\right)^2} \begin{matrix} \le \\ \ge \end{matrix} 0 \quad \text{for} \quad c_t^{(1)} \begin{matrix} \ge \\ \le \end{matrix} 0 \quad \text{and}$$

$$\frac{\partial u\left(\bar{c}_t^{(0)}, c_t^{(1)}\right)}{\partial c_t^{(1)}} < \frac{\partial u\left(\bar{c}_t^{(0)}, -c_t^{(1)}\right)}{\partial c_t^{(1)}} \quad \text{for} \quad c_t^{(1)} \ge 0. \tag{13}$$

[14]

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[13] That is, the level of consumption in the previous period is regarded as the reference point and it is renewed every period.

[14] The expression is based on that of Tversky and Kahneman (1992) though it has an inconsistency at $c_t^{(1)} = 0$.

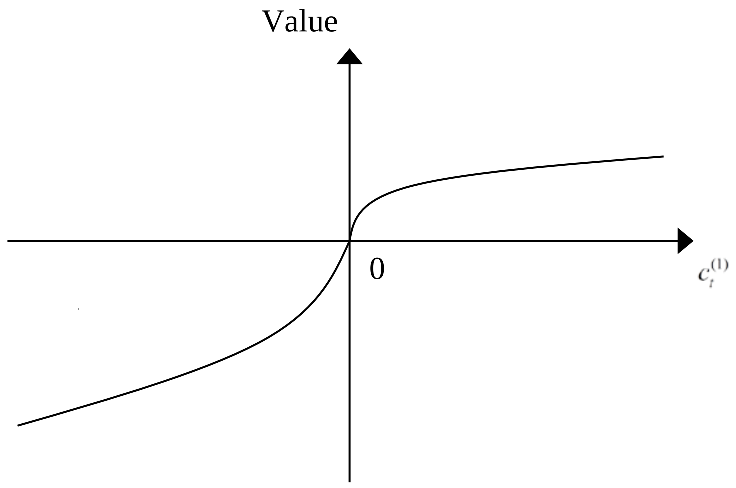

Figure 6.1 Value Function of Kahneman and Tversky

A Simple Case

Noting (13), specify the utility function as follows:[15][16]

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[15] Some behavioral economists strictly distinguish "utility function" from "value function." Note that this study is using the terminology "utility" in a broader sense.

[16] Other specifications may include the ones such as

$$u_t = \begin{cases} \alpha_0 \cdot \left(c_t^{(0)}\right)^{\beta_0} + \alpha_{1+}\left(\left|c_t^{(1)} + 1\right|^{\beta_{1+}} - 1\right) & for \quad c_t^{(1)} \geq 0 \\ \alpha_0 \cdot \left(c_t^{(0)}\right)^{\beta_0} + \alpha_{1-}\left(\left|c_t^{(1)} - 1\right|^{\beta_{1-}} - 1\right) & for \quad c_t^{(1)} < 0. \end{cases}$$

The reason of having a unique $\left|c_t^{(1)} + 1\right|^{\beta_{1+}}$ or $\left|c_t^{(1)} - 1\right|^{\beta_{1-}}$ and the above expression (or a common β in the text) is due to the fact that:

For $\beta_1 > \beta_1'$, $x^{\beta_1} \underset{<}{\geq} x^{\beta_1'}$ as $x \underset{<}{\geq} 1$.

That is, the relative values are reversed around $x = 1$.

For the third and fourth lines of (13) to be satisfied, (for one function to be always larger than the other,) $x \geq 1$ must be guaranteed. Also, it is desirable for the values of the two functions to coincide with each other at $c_t^{(1)} = 0$. Thus, 1 is subtracted from the expressions.

$$u_t = \begin{cases} \alpha_0 \cdot \left(c_t^{(0)}\right)^{\beta_0} + \alpha_{1+}\left|c_t^{(1)}\right|^{\beta_1} & for \quad c_t^{(1)} \geq 0 \\ \alpha_0 \cdot \left(c_t^{(0)}\right)^{\beta_0} + \alpha_{1-}\left|c_t^{(1)}\right|^{\beta_1} & for \quad c_t^{(1)} < 0. \end{cases} \tag{14}$$

$\alpha_{1+} > 0$, $\alpha_{1-} < 0$, $|\alpha_{1-}| > |\alpha_{1+}|$,

$0 < \beta_1 < 1$.

First, investigate the case in which $\alpha_0 = 0$ and $r = \rho = 0$. Then, if the utility function is concave (as usual,) the solid curve or straight line in Figure 4.2 characterizes the optimal path. However, if the utility has a convex part, there can be another type of solution.

Consider reducing $c_t^{(1)}$ of some period and raising those of the other periods. Note that, as long as $c_0^{(0)}$ and $c_T^{(0)}$ are fixed, the sum of the new changes must be zero. Since the utility function is convex in the negative domain, as $c_t^{(1)}$ is reduced, its marginal effect declines. If the corresponding amount of $c_t^{(1)}$ is distributed among many other periods, the life-time utility may rise because there are only small changes in the marginal effects on the recipient side: An individual receives more daily bread in exchange for extraordinary misery in some period.

Figure 6.2 illustrates the idea more clearly. For simplicity, suppose a_0 is sufficiently large so that the (life-time) income constraint does not bind ($\mu = 0$.) Then, without convexity, $c_t^{(1)*} = \left(c_T^{(0)} - c_0^{(0)}\right)/T$ constitutes the best path of $c_t^{(1)}$. When $c_{\hat{t}}^{(1)}$, $c_t^{(1)}$ of period $\hat{t}$, is reduced by $\Delta c^{(1)}$, the utility of that period decreases

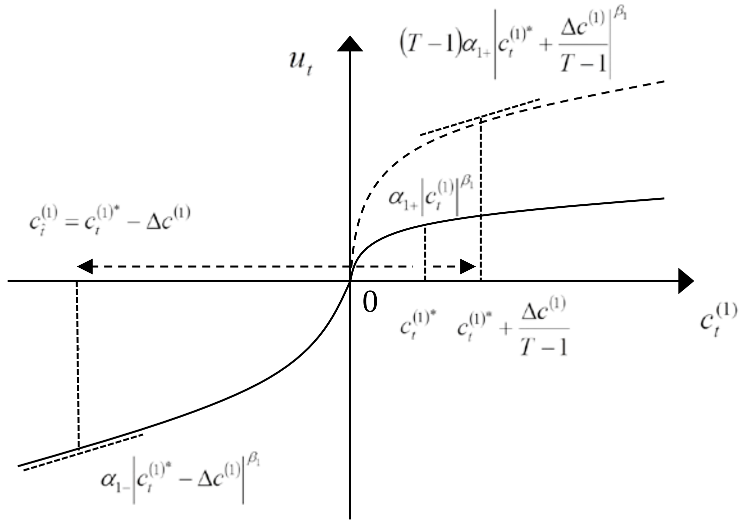

Figure 6.2

from $\alpha_{1+}\left|c_t^{(1)*}\right|^{\beta_1}$ to $\alpha_{1-}\left|c_t^{(1)*}-\Delta c^{(1)}\right|^{\beta_1}$. However, by raising $c_t^{(1)}$ of the other periods by $\Delta c^{(1)}/(T-1)$, each increases $\alpha_{1+}\left|c_t^{(1)*}\right|^{\beta_1}$ from to $\alpha_{1+}\left|c_t^{(1)*}+\Delta c^{(1)}/(T-1)\right|^{\beta_1}$. If the slope in the negative domain is flatter enough and T is sufficiently large, the life-time utility rises.

Next, consider more general case: $\alpha_0, \alpha_1 > 0$. If the absolute level of consumption is also accounted for, the (life-time) income constraint always binds ($\mu > 0$.) Then, the choice the period in which $c_t^{(1)}$ is reduced is also crucial. Reducing $c_t^{(1)}$ in earlier period(s) is typically better because it saves the asset.

Figure 6.3 indicates the constrained and unconstrained optimal paths of $c_t^{(0)}$ calculated numerically for $T=7$, $r=\rho=0$, $\alpha_0=0.1$, $\alpha_{1+}=0.5$, $\alpha_{1-}=-0.6$, $\beta_0=\beta_1=0.5$, $c_0^{(0)}=100$, $c_7^{(0)}=300$ and $a_0^{(0)}=1000$.

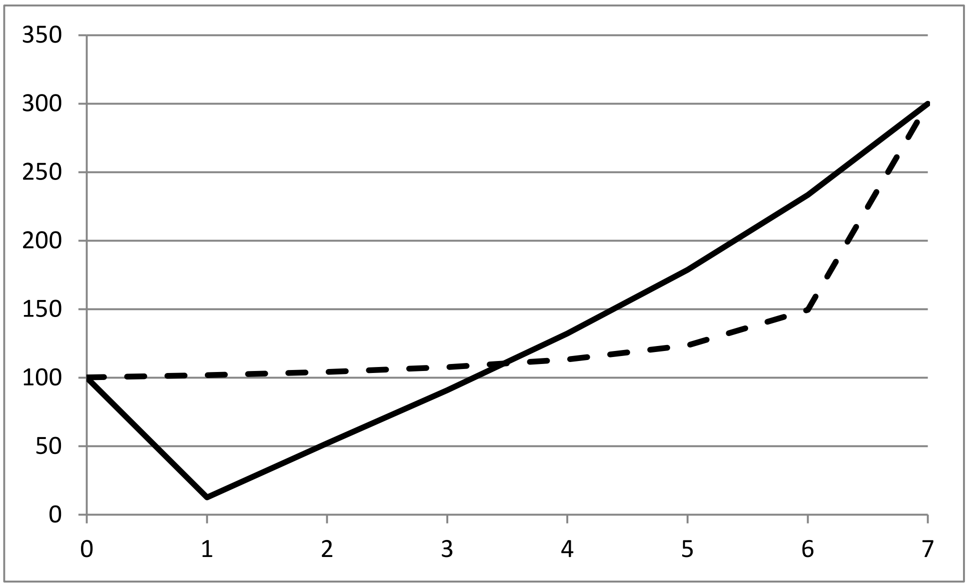

Figure 6.3 Comparing Paths of $c_t^{(0)}$

While the broken line corresponds to the optimal path when $c_t^{(1)}$ is constrained to be positive (i.e. only the concave part of the utility function is effective,) the solid line indicates the unconstrained one (i.e. the convex part of the utility function can be used.) The life-time utility of the former is 22.02 and that of the latter is 22.83. The latter profile exhibits a sharp decline in the first period and rises in the other periods. That is, “work now, play later” is the best for him/her.

7. Capital Accumulation

In the preceding sections, it has been assumed that an individual is endowed with some fixed amount of asset. This section extends the model to consider capital accumulation and resulting economic growth.

A model of capital accumulation does not usually characterize the decision-making of an individual but a country. Thus, there are several points to be noted before proceeding to the analysis: 1) An individual lives for finite periods but a country (or a family,) which consists of individuals with a bequest motive or any altruistic consideration, may last forever. 2) As has been argued so far, an individual may have a “terminal condition” on his/her consumption while a country usually does not, even if the planning horizon is finite. 3) Basic (domestic) model of capital accumulation does not allow a country to borrow or lend. As a result, it needs to develop on its own and the interest rate is endogenously determined. However, an individual can usually borrow or lend (at market rate) and does not need to be perfectly independent from the society.

Despite those concerns, investigating the model with capital accumulation is of theoretical interest. Then, one may ask: How the outcome differs from the traditional one when the change in consumption matters?

Throughout this section, consider not a Kahneman Tversky-type but a simple concave utility function. Suppose that the production is described by a Cobb-Douglas function of capital $K_t^{(0)}$ and labor L_t in each period:

$$Y_t = \gamma_0 \left(K_t^{(0)}\right)^{\gamma_k} \left(L_t\right)^{1-\gamma_k} \tag{15}$$

It is either consumed or invested: [17]

$$Y_t = C_t^{(0)} + K_{t+1}^{(1)} \tag{16}$$

where $K_{t+1}^{(1)} \equiv K_{t+1}^{(0)} - K_t^{(0)}$

In per-capita terms, assuming the growth rate *n* of population (labor) is constant, (16) can be rewritten as:

$$y_t = \gamma_0 \left(k_t^{(0)}\right)^{\gamma_k} = c_t^{(0)} + k_{t+1}^{(1)} + nk_{t+1}^{(0)} \tag{17}$$

where $y_t \equiv Y_t / L_t$, $c_t^{(0)} \equiv C_t^{(0)} / L_t$, $k_t^{(0)} \equiv K_t^{(0)} / L_t$, $k_{t+1}^{(1)} \equiv k_{t+1}^{(0)} - k_t^{(0)}$,

and $n = \dfrac{L_{t+1} - L_t}{L_t}$.

The planner maximizes the (discounted) sum of the utilities of a representative individual over periods $1, \cdots, T$ and the value of capital remaining in $T+1$, measured by asset function ϕ:[18] Using (2),

---

[17] (15) and (16) can be written as $Y_t = \gamma_0 \left(K_{t-1}^{(0)}\right)^{\gamma_k} \left(L_t\right)^{1-\gamma_k}$ and $Y_t = C_t^{(0)} + K_t^{(1)}$, respectively, to be consistent with (2). However, taking into account common macroeconomic notations, time suffix of capital (asset) is redefined in this section.

[18] The functional form is specified later in this section. Without the asset function, the economy has no incentive to accumulate capital in later periods and zero or negative investments are to be seen.

$$\sum_{t=1}^{T}\left[\left(\frac{1}{1+\rho}\right)^{t-1}\left\{\alpha_0\left(c_t^{(0)}\right)^{\beta_0}+\alpha_1\left(c_t^{(1)}+\tilde{c}^{(1)}\right)^{\beta_1}\right\}\right]+\left(\frac{1}{1+\rho}\right)^{T}\phi\left(k_{T+1}^{(0)}\right) \tag{18}$$

Noting (17), the Lagrangian for this problem is,

$$\begin{aligned} L=&\sum_{t=1}^{T}\left[\left(\frac{1}{1+\rho}\right)^{t-1}\left\{\alpha_0\left(c_t^{(0)}\right)^{\beta_0}+\alpha_1\left(c_t^{(1)}+\tilde{c}^{(1)}\right)^{\beta_1}\right\}\right]+\left(\frac{1}{1+\rho}\right)^{T}\phi\left(k_{T+1}^{(0)}\right)\\ &+\sum_{t=1}^{T}\lambda_t^{c(1)}\left\{\left(c_t^{(0)}-c_{t-1}^{(0)}\right)-c_t^{(1)}\right\}\\ &+\sum_{t=2}^{T+1}\lambda_t^{k(1)}\left\{\left(k_t^{(0)}-k_{t-1}^{(0)}\right)-k_t^{(1)}\right\}\\ &+\sum_{t=1}^{T}\lambda_t^{y}\left\{\gamma_0\left(k_t^{(0)}\right)^{\gamma_k}-\left(c_t^{(0)}+k_{t+1}^{(1)}+nk_{t+1}^{(0)}\right)\right\}. \end{aligned} \tag{19}$$

The first-order conditions are:

$$\frac{\partial L^*}{\partial c_t^{(0)}}=\left(\frac{1}{1+\rho}\right)^{t-1}\alpha_0\beta_0\left(c_t^{(0)}\right)^{\beta_0-1}+\lambda_t^{c(1)}-\lambda_{t+1}^{c(1)}-\lambda_t^{y}=0\ ,$$

$$\text{for}\quad t=1,\cdots,T-1, \tag{20}$$

$$\frac{\partial L^*}{\partial c_T^{(0)}}=\left(\frac{1}{1+\rho}\right)^{T-1}\alpha_0\beta_0\left(c_T^{(0)}\right)^{\beta_0-1}+\lambda_T^{c(1)}-\lambda_T^{y}=0\ ,\ \text{[19]} \tag{21}$$

$$\frac{\partial L^*}{\partial c_t^{(1)}}=\left(\frac{1}{1+\rho}\right)^{t-1}\alpha_1\beta_1\left(c_t^{(1)}+\tilde{c}^{(1)}\right)^{\beta_1-1}-\lambda_t^{c(1)}=0,\quad\text{for}\quad t=1,\cdots,T\ , \tag{22}$$

$$\frac{\partial L^*}{\partial k_t^{(0)}}=\lambda_t^{k(1)}-\lambda_{t+1}^{k(1)}+\lambda_t^{y}\gamma_0\gamma_k\left(k_t^{(0)}\right)^{\gamma_k-1}-n\lambda_{t-1}^{y}=0\ ,$$

$$\text{for}\quad t=2,\cdots,T\ , \tag{23}$$

$$\frac{\partial L^*}{\partial k_{T+1}^{(0)}}=\left(\frac{1}{1+\rho}\right)^{T}\frac{d\phi\left(k_{T+1}^{(0)}\right)}{dk_{T+1}^{(0)}}+\lambda_{T+1}^{k(1)}-n\lambda_{T-1}^{y}=0\ , \tag{24}$$

[19] In this section, $c_T^{(0)}$ is assumed to be variable.

$$\frac{\partial L^*}{\partial k_t^{(1)}} = -\lambda_t^{k(1)} - \lambda_{t-1}^y = 0 \ , \qquad \text{for} \quad t = 2,\cdots,T+1, \qquad (25)$$

The only difference of (20) and (22) from conditions (5) and (6) is that $\mu\{1/(1+r)\}^{t-1}$ is replaced by λ_t^y. By (23) and (25),

$$\lambda_t^y = \frac{1+n}{1+r_t}\lambda_{t-1}^y \qquad \text{for} \quad t = 2,\cdots,T\ , \qquad (26)$$

where $r_t = \gamma_0\gamma_k\left(k_t^{(0)}\right)^{\gamma_k-1}$.

Note that r_t decreases as $k_t^{(0)}$ grows. If initial $k_0^{(0)}$ is sufficiently small, $\lambda_t^y < \lambda_{t-1}^y$ as long as $r_t > n$.

Comparing the Cases With and Without the Utility of Change

Finally, compare the consumption path derived from the above conditions with the traditional one.

i) Shape of the path of $c_t^{(0)}$

The modified golden rule of this problem is $1 + r_t^* = 1 + \gamma_0\gamma_k\left(k^{(0)*}\right)^{\gamma_k-1} = (1+n)(1+\rho)$. Suppose $k_0^{(0)}$ is sufficiently small so that the steady state is not reached within the planning horizon. Then, if $\alpha_1 = 0$ (the change does not matter,) by (20), (21) and (26), $c_t^{(0)}$ is increasing in t.[20]

[20] Concavity or convexity of $c_t^{(0)}$ is not generally asserted by the conditions.

For $\alpha_1 > 0$, similarly to that of (the latter part of) Section5, show that if $c_{\hat{t}}^{(1)} < 0$ and $c_{\hat{t}+1}^{(1)} \geq 0$ for some $\hat{t}$, then $c_t^{(1)} < 0$ for all $t \leq \hat{t}$.

By (20) and (21),

$$
\begin{aligned}
&(1+\rho)^{\hat{t}-2}\lambda_{\hat{t}-1}^{y} \\
&= \frac{1+r_{\hat{t}}}{(1+n)(1+\rho)}\left\{\alpha_0\beta_0\left(c_{\hat{t}}^{(0)}\right)^{\beta_0-1} + \alpha_1\beta_1\left(c_{\hat{t}}^{(1)} + \tilde{c}^{(1)}\right)^{\beta_1-1} - \frac{1}{1+\rho}\alpha_1\beta_1\left(c_{\hat{t}+1}^{(1)} + \tilde{c}^{(1)}\right)^{\beta_1-1}\right\} \\
&= \alpha_0\beta_0\left(c_{\hat{t}-1}^{(0)}\right)^{\beta_0-1} + \alpha_1\beta_1\left(c_{\hat{t}-1}^{(1)} + \tilde{c}^{(1)}\right)^{\beta_1-1} - \frac{1}{1+\rho}\alpha_1\beta_1\left(c_{\hat{t}}^{(1)} + \tilde{c}^{(1)}\right)^{\beta_1-1}.
\end{aligned}
$$

$$\text{for} \quad \hat{t} = 2, \cdots, T-1, \quad (27)$$

If $1+r_t > (1+n)(1+\rho)$ for all *t*, by the same logic used in the latter part of Section 5, $c_0^{(0)} > c_1^{(0)} > c_2^{(0)} > \cdots > c_{\hat{t}-1}^{(0)} > c_{\hat{t}}^{(0)}$. That is, $c_t^{(0)}$ has at most one local minimum.

ii) Relative position

Show that the path of $c_t^{(0)}$ drawn when $\alpha_1 > 0$ intersects its original ($\alpha_1 = 0$) counterpart at least once.

Suppose not. Then, one of the following is the case: 1) the former coincides with the latter in all periods, 2) the former is higher than the latter in at least one period and higher than or equal to the latter in any other periods or 3) the former is lower than the latter in at least one period and lower than or equal to the latter in any other periods.

It can be easily confirmed that the first-order conditions are not satisfied if 1) is the case. As for 2), the latter ($\alpha_1 = 0$) path is inefficient because the former ($\alpha_1 > 0$) is feasible and always gives better (or at leat equal) $c_t^{(0)}$s even when $\alpha_1 = 0$. And it can be shown that 3) is not possible as follows:

(17) can be rewritten to be:

$$k_{t+1}^{(0)} = \frac{1}{1+n}\left\{\gamma_0\left(k_t^{(0)}\right)^{\gamma_k} - nk_t^{(0)} - c_t^{(0)}\right\} + k_t^{(0)} \tag{28}$$

Thus, by reducing $c_t^{(0)}$, $k_{t+1}^{(0)}$ increases. When $k_{t+1}^{(0)}$ increases while $c_{t+1}^{(0)}$ is not raised, $k_{t+2}^{(0)}$ also increases because:

$$\frac{dk_{t+2}^{(0)}}{dk_{t+1}^{(0)}} = \frac{1+r_2}{1+n} \tag{29}$$

As a result, $k_T^{(0)}$ is larger than the original ($\alpha_1 = 0$) level. It can be easily confirmed that $c_T^{(0)}$ must be larger. A contradiction.

Figure 7.1 illustrates the optimal paths of $c_t^{(0)}$ calculated numerically for the cases of $\alpha_1 = 0$ (broken line) and $\alpha_1 = 0.005$ (solid line) when $T = 7$, $\rho = 0.01$, $n = 0.02$, $\alpha_0 = 0.5$, $\beta_0 = \beta_1 = 0.5$, $\tilde{c}^{(1)} = 1$, $\gamma_0 = 5$, $\gamma_0 = 0.4$, $c_0^{(0)} = 34.976$, $k_1^{(0)} = 800$. The asset function is specified as:[21]

$$\phi\left(k_t^{(0)}\right) \equiv \frac{1+\rho}{\rho}\alpha_0\left\{\gamma_0\left(k_t^{(0)}\right)^{\gamma_k} - nk_t^{(0)}\right\}^{\beta_0 - 1}.$$

[21] The asset function is defined so as to make the present value of the flow of steady-state level of consumption coincide with the value of the steady-state level of capital. In the setting of this section, the steady-state levels of consumption and capital are calculated to be $c^{(0)*} = 60.163$ and $k^{(0)*} = 1{,}084.023$, respectively.

The solid line represents a typical case in which an individual saves (and invests) more in earlier periods and raises consumption in later periods if α_1 is positive.

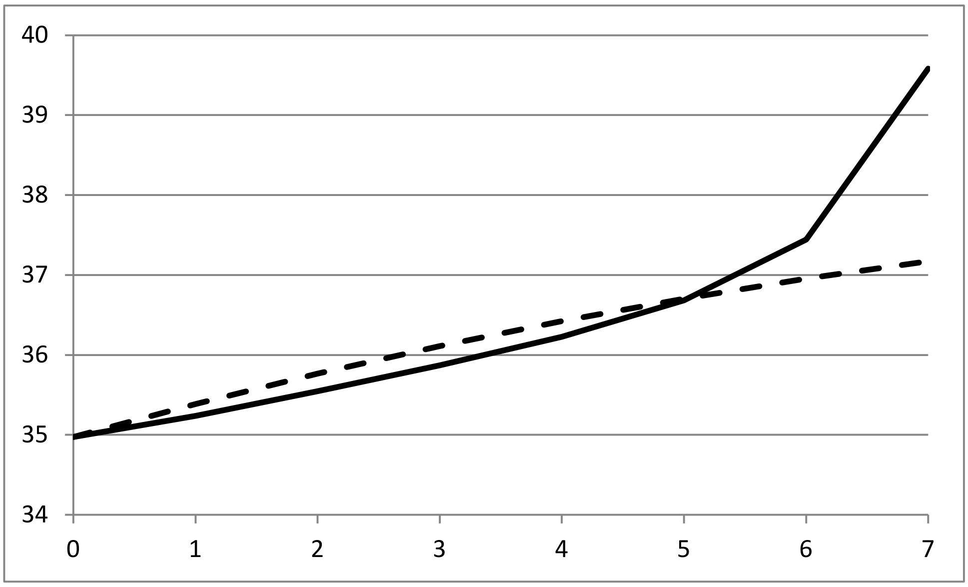

Figure 7.1 Capital Accumulation and Optimal Paths of $c_t^{(0)}$

Conclusion

This study has characterized the behavior of an individual who values the change in his/her consumption. It has been shown that, although an individual tends to seek an upward-sloping consumption path, if there are initial and terminal conditions, the overall shape of the path depends highly on them. When an individual is concerned with both the (first-order) change and the absolute level, the path is concave when the amount of initial asset is relatively large, convex when it is relatively small and has both concave and convex parts when the amount is in between them. In the case utility function is of Kahneman and Tversky-type, sharp decline in an earlier period, followed by gradual increases, typically results.

Further, the case in which capital accumulation is taken into account has been investigated. For initial level of capital lower than that of steady state, if the change in consumption matters, relatively larger saving (investment) in earlier periods and higher consumption in later periods typically take place.

Extensions of this study may include utility as a function of asset and as a function of the *rate* of the change in consumption or the second (and higher) order changes. Also, the levels of life-time satisfactions can be compared for *given* typical consumption paths. Moreover, any related empirical studies can be performed.

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