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Abstract

This paper deals with a variable spatial-distribution of consumers according to the location decisions of firms in spatial competition. Specifically, we present a location-then-quantity game in a circular city in which some of the consumers are attracted to the firms' locations. We show that with this scenario, all firms agglomerate when the transport cost is low. This is in sharp contrast to the results shown in previous studies with fixed distributions of consumers, where such a full agglomeration never occurs in equilibrium. Welfare analysis shows excess dispersion compared with the second-best scenario. By changing the space into a line segment, we show that a spatial dispersion can be achieved.

JEL Classification: L11; L13; R12

Keywords: Spatial Cournot competition; Mobile consumers; Transport cost

1 Introduction

The study of spatial competition has a long history, beginning with the seminal work by Hotelling (1929). Despite the accumulation of knowledge, there is an unexplored issue: variable distribution of consumers. In the research about spatial competition, the spatial distribution of consumers is exogenously fixed throughout the analysis. The distribution could vary, however, after firms

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determined their locations. Typically, firms attract some consumers to (or near) their locations for the following reasons.[1]

1. Advertisement, awareness of the products

   If firms advertised their products, some consumers would become aware of the products' merits. We can interpret this to mean that the effective mass of consumers attracted to each firm's characteristic is enhanced.

2. Network externalities

   If the good generates network externalities (network good), it is important for consumers that other consumers also own the same product. In other words, consumers prefer a prevalent product to a brand-new, different type of product that few people use. This preference also means there will be an enhanced mass of consumers of the established products.

3. Shopping externalities, comparison or one-stop shopping

   Suppose that there are other goods that are outside of the model. For instance, we analyze a fish market in the model and a meat market is behind the model. If consumers buy both the fish and the meat, the locations where both goods are available have a great advantage for consumers who want to do comparison shopping or one-stop shopping to minimize their shopping costs. Hence, consumers prefer such locations, and the population there grows faster than at other locations.[2]

4. Mobile workers (employees)

   Suppose that some of the consumers are employed by the firms, and their commuting costs are not negligible. In this case, they would love to live at or near the employing firms. If a firm changes its location from town A to town B, the purchasing power at town A shrinks, while that at town B increases, because of the job disappearance and creation,

[1] If firms generated negative externalities toward their neighbors, such as contaminated smoke, they would be avoided by consumers. Such an opposite scenario is less interesting in our model, as seen later. That is, our benchmark model already yields spatial dispersion of firms; hence, the outcome is to be unchanged with our new element.

[2] See, e.g., Stahl (1982) and Wolinsky (1983) for consumers' search and the concentration of retail firms as a related issue.

respectively. Therefore, the population (mass of demand) increases at the firms' locations. Due to the multiplier effect, such job creation is important for the local economy. For example, Japanese automaker Toyota Motor Corporation announced in March 2010 that their production operation would be moved from California to Mississippi, which means 2,000 new jobs should be created in Mississippi. Haley Barbour, the governor of Mississippi, welcomed the decision, whereas strong opposition occurred in California.

Given these reasons why consumers are attracted to firms' locations, the objective of the present paper is to develop a model of spatial competition that deals with such a variable distribution of consumers according to firms' locations. For expository convenience, let the term *"mobile consumers"* denote those who change their locations after firms choose their locations. Specifically, we assume that a constant population (mass) of mobile consumers always clings to the firms. In other words, to simplify the analysis we ignore an endogenous residential-choice problem for mobile consumers throughout most of our discussion (we will discuss this issue in Section 5).

Our analysis is based on spatial Cournot competition with homogeneous good that was developed by Hamilton *et al.* (1989) and Anderson and Neven (1991),[3] in which oligopolistic firms choose their locations simultaneously in the first stage, and they decide their supply amount simultaneously for (immobile) consumers that are distributed over the space. On the one hand, their studies both show that, in such a linear space, the firms agglomerate in the centre in equilibrium. On the other hand, Pal (1998) shows that, in a circular city, two firms are located as far away from each other as possible on the circumference (e.g., at 12 a.m. and at 6 a.m.). In sum, the results depend greatly on the form of the space, linear or circular.

In our model, in addition to immobile consumers, mobile consumers are allocated at firms' locations, and each firm serves all consumers.[4] The introduction of mobile consumers induces additional agglomeration and dispersion forces. If a firm chooses a location near a rival, there

[3] Another branch of spatial competition is spatial Bertrand (price) competition (e.g., d'Aspremont *et al.,* 1979). In the present paper we adopt the Cournot setting, because it brings us clearer and more contrastive results.

[4] Gupta *et al.* (1997) extend the consumer distribution to more general cases. However, the distribution is exogenously fixed to the end.

is a positive effect in the sense that the profits would increase due to lower transport costs for the mobile consumers who normally patronize the rival (agglomeration force). Meanwhile, the original firm should lose profits, because its mobile consumers are attracted by the approaching rival (dispersion force). As a result, we find that *agglomerated equilibrium is achieved even in a circular city when the transport cost is sufficiently low, whereas dispersed equilibrium is present even in a linear city when the transport cost is high.* In other words, the form of the space is not so determinative. This is in sharp contrast to the familiar results mentioned above (Hamilton *et al.,* 1989; Anderson and Neven, 1991; Pal, 1998).[5]

When it comes to mobile consumers, the *new economic geography* (henceforth, NEG) launched by Krugman (1991) deals with a similar issue of spatial distribution of economic activities in general equilibrium incorporating scale economies in production and imperfect competition (specifically, monopolistic competition).[6] The findings of the NEG models and ours are somewhat similar: For instance, lower transport cost tends to lead to agglomerated equilibrium (core-periphery structure). Nevertheless, in monopolistic competition of the NEG, each firm has only a negligible impact on the economy, which does not match our awareness of the issues. Despite the difference between the NEG and our oligopoly, Picard and Tabuchi (2010) developed a model similar to ours: They present a general equilibrium model à la NEG over a circular, continuous space. In their model, the flat earth equilibrium (uniform distribution of firms) is unstable in a wide class of transport costs. Therefore, the NEG structure puts more stress on the agglomeration force (e.g., forward and backward linkages) than does our partial equilibrium in an oligopoly. In our model, agglomeration is not so robust.

The remainder of the paper is organized as follows. In Section 2, the two-stage location-quantity game is presented. In Section 3, the quantity choices are analyzed. In Section 4, the location equilibrium is established. Section 5 is devoted to welfare analysis. Section 6 deals with some extensions of our model. Section 7 summarizes the results.

[5] Matsushima (2001), Shimizu and Matsumura (2003) and Matsumura and Matsushima (forthcoming) all show that full agglomeration never occurs in equilibrium in a circular city with more than two firms.

[6] See Fujita *et al.* (1999) for an excellent comprehensive survey of NEG.

2 The model

Our benchmark is a spatial Cournot model with the circular city developed by Pal (1998). There is a circular city with perimeter equal to 1. There are two firms indexed by i ($i = 1, 2$) that supply a homogeneous good with zero marginal cost, and let $x_i \in [0, 1)$ denote the location of firm i.

There are two types of consumers, mobile and immobile. Immobile consumers are uniformly and continuously distributed on $[0, 1)$ with a unit density at each location, while mobile consumers are able to move freely with zero relocation cost in the city after the firms determine their locations. As mentioned in the Introduction, we assume that mobile consumers are attracted to firms, and each mobile consumer immediately moves to either of the firms' locations. To simplify the analysis, the movement is exogenous. Moreover, because the firms are symmetric, each firm attracts the same number (mass) of mobile consumers.[7] Let $n > 0$ be the mass of mobile consumers attracted by each firm, and let $f(z)$ be the density function of mobile consumers at z. Thus, we have

$$f(z) = \begin{cases} 0 & \text{if } z \neq x_1, x_2 \\ n & \text{if } z = x_i \neq x_j,\ i \neq j \\ 2n & \text{if } z = x_1 = x_2 \end{cases} . \tag{1}$$

The third case is referred to as *full agglomeration* in the sense that all firms agglomerate at a point.

Each consumer, irrespective of the types, has the same, inverse demand function as follows:

$$P = a - bQ, \quad Q = q_1 + q_2, \tag{2}$$

where P is the price, q_i is firm i's supply amount, and a and b are constants.

We consider a two-stage location-then-quantity game. In the first stage, the firms choose their locations simultaneously. In the second stage, they determine their supply amounts simultaneously after the mobile consumers move. Subgame perfection is adopted as the equilibrium concept.

[7] See Section 5 for the discussion of endogenous residential choice and Section 6 for the asymmetric cases.

The firms bear transport costs, and they can set an independent supply amount for each location, because arbitrage between consumers is prohibitively costly. To ship a unit of the product from x_i to a consumer at z, the transport cost for firm i is given by $T(x_i, z) = \min\{t|x_i - z|, t(1 - |x_i - z|)\}$, where $t > 0$ is the transport cost parameter and is assumed to be sufficiently low, such that

$$0 < \tau \equiv t/a < 1. \tag{3}$$

This requirement ensures that both firms serve the entire city (all consumers), irrespective of the locations. Let

$$\pi_i(z) = q_i(z) \left[P(z) - T(x_i, z)\right] \tag{4}$$

denote the local profit for firm i earned from a consumer at z. Then, the total profit is given by

$$\Pi_i = \int_0^1 \pi_i(z)dz + n\pi_i(x_i) + n\pi_i(x_j) \quad \text{for } i, j \in \{1, 2\},\ i \neq j, \tag{5}$$

where the first term is the profits earned from immobile consumers, and the second and the third terms are those from the mobile consumers.

3 Quantity equilibrium

Let us analyze the second-stage Cournot competition due to backward induction. First, we consider immobile consumers. Recall that each local market is independent. Then, the first-order condition $\partial\pi_i(z)/\partial q_i(z) = 0$ yields the equilibrium quantity for firm i at z as follows (the asterisk refers to the equilibrium value):

$$q_i^*(z) = \left[a - 2T(x_i, z) + T(x_j, z)\right]/3b \quad \text{for } i, j \in \{1, 2\},\ i \neq j. \tag{6}$$

Next, we consider mobile consumers. The first-order conditions $\partial\pi_i(x_i)/\partial q_i(x_i) = 0$ and $\partial\pi_i(x_j)/\partial q_i(x_j) = 0$ yield the equilibrium quantities for the mobile consumers at x_i and x_j as follows:

$$q_i^*(x_i) = [a - 2T(x_i, x_i) + T(x_j, x_i)]/3b = (a + T(x_j, x_i))/3b, \quad (7)$$

$$q_i^*(x_j) = [a - 2T(x_i, x_j) + T(x_j, x_j)]/3b = (a - 2T(x_i, x_j))/3b \quad (8)$$

for $i, j \in \{1, 2\}, i \neq j$. Substituting (6), (7), and (8) into (4), the local profits are rewritten as $\pi_i^*(z) = b[q_i^*(z)]^2$, $\pi_i^*(x_i) = b[q_i^*(x_i)]^2$, and $\pi_i^*(x_j) = b[q_i^*(x_j)]^2$, respectively. Then, from (5), the total profit is obtained by

$$\Pi_i^*(x_i) = \int_0^1 \pi_i^*(z)dz + n\pi_i^*(x_i) + n\pi_i^*(x_j). \quad (9)$$

It is noteworthy that the profit from the firm's own mobile consumers, $\pi_i^*(x_i)$, increases by separating from the rival, whereas the profit from the rival's mobile consumers, $\pi_i^*(x_j)$, increases by approaching the rival. This tradeoff is the key to determining the location equilibrium.

4 Location equilibrium

Here, we analyze the location equilibrium[8] in the first stage when the quantity choices are given in the second stage, as described in Section 3. Let us first consider the best response of firm 1 against a given location of firm 2. Due to the symmetry with regard to the space, we can assume without loss of generality that $0 \leq x_1 \leq 1/2$ and $x_2 = 0$. Let x_1^* be the best response against $x_2 = 0$. In other words, x_1^* is firm 1's optimal (profit-maximizing) distance from firm 2. Meanwhile, because of the symmetry with regard to the firms, firm 2's optimal distance from firm 1 must be also x_1^*. This situation constitutes an equilibrium for any location pair such that the distance between the firms is x_1^*.[9]

Fortunately, the profit function given by (9) takes a simple form that allows an analytical solu-

[8] A location pair (x_1^*, x_2^*) is a Nash equilibrium if and only if $\Pi_i^*(x_i^*) \geq \Pi_i^*(x_i) \quad \text{for } \forall i \text{ and } \forall x_i \in [0, 1)$.
[9] If x_1^* were multivalued, multiple equilibria could arise as discussed later.

tion: It is cubic with regard to x_i and the coefficient of x_i^3 is negative. Tedious but straightforward calculations yield the following main result:

Proposition 1 *Let*

$$T(n) = \begin{cases} 64n/3(5n+2)^2 & \text{when } 0 < n \leq 2/15 \\ 12n/(15n+2) & \text{when } 2/15 < n \end{cases}. \tag{10}$$

The location equilibrium is classified into three groups as follows:

(i) Full agglomeration (minimal differentiation): Two firms agglomerate at a point if $\tau \leq T(n)$*.*

(ii) Full dispersion (maximal differentiation): Two firms separate from each other such that the distance between them is equal to $1/2$ *if* $\tau \geq \max\{2/5, T(n)\}$*.*

(iii) Partial dispersion (in-between differentiation): Two firms separate from each other such that the distance between them is less than $1/2$ *if* $T(n) \leq \tau < 2/5$ *under* $0 < n < 2/15$*. The distance is given by* $\left(5n + 2 + \sqrt{(5n+2)^2 - 16(n/\tau)}\right)/8$*.*

Proof. See Appendix A. ■

[Insert Figure 1 here.]

[Insert Figure 2 here.]

Figure 1 classifies the location equilibria in the parameter space. Because $T(n)$ is increasing in n, agglomeration is likely to occur when the mass of mobile consumers is large (large n) or the transport costs are low (small τ) or both. Figure 2 shows a pitchfork diagram of the location equilibrium when $n = 1/10$ with the normalization of $x_1 + x_2 = 1$. Unlike Pal (1998) with $n = 0$, $T(n) > 0$ for any positive n, which implies that sufficiently small transport costs generate full agglomeration. In sum, *full agglomeration always exists, even in the circular city.* This is in sharp contrast to the model described by Pal (1998).

It is true that a large n tends to lead to agglomeration. Nonetheless, because $T(n)$ is bounded from above ($\lim_{n\to\infty} 12n/(15n+2) = 4/5$), full dispersion always exists as an equilibrium, even when n is large.[10] Recall the total profits in (9). The function degenerates into $\Pi_i^*(x_i) = n \times [\pi_i^*(x_i) + \pi_i^*(x_j)]$ when n is too large.[11] In other words, only the sales to mobile consumers matter. Because n is just a multiplier of the function, n is independent of the determination of location equilibrium. Accordingly, the profits earned from the mobile consumers, $\pi_i^*(x_i) + \pi_i^*(x_j)$, is determinative for the equilibrium. Let us take a closer look to understand of our model.

It is helpful to borrow the ideas from a model of Cournot duopoly with asymmetric cost.[12] Suppose that $P = 1 - Q$ as the inverse demand function, and the marginal cost of a firm (superior firm) is zero and that of the other firm (inferior firm) is $c > 0$. Then, the sum of the (equilibrium) profits of the two firms corresponds to $\pi_i^*(x_i) + \pi_i^*(x_j)$ above, and the location choice in our model corresponds to the choice of c (the argument is essentially the same by setting $c = \tau/2$). Tedious calculations yield the sum of the profits of the two firms as $(2 - 2c + 5c^2)/9$, which is a parabola whose minimum is given by $c = 1/5$. On the one hand, when $c > 1/5$, that sum of the profits is increasing in c and is maximized at $c = 1/2$ (monopolization). This suggests that when the transport costs are high, the firms will choose location dispersion in search of (local) monopoly. On the other hand, when $c < 1/5$, the sum of the profits is decreasing in c and is maximized at $c = 0$ (cost reduction). Therefore, when the transport costs are low, the firms will agglomerate in search of strong competitiveness against the rival.[13] Furthermore, immobile consumers are evenly spread over the space; hence, they work as a dispersion force, as Pal (1998) has shown. To synthesize the descriptions of these effects, we present Proposition 1.

Note that there is a possibility of multiple equilibria. For instance, when $n = 1/2$ and $\tau = T(1/2) = 12/19$, both full agglomeration and full dispersion are equilibria. The multiplicity of equilibria, however, is restricted to a zero-measure set of parameter values. Hence, location

[10] The no-black-hole condition, which is often assumed in NEG, always holds in our model. In other words, agglomeration forces here are weaker than in NEG.

[11] We can also interpret the case as the model without immobile consumers.

[12] See, among others, Boyer *et al.* (2003) and Ziss (1993) for spatial competition with asymmetric cost.

[13] See Kabiraj and Marjit (2000) for the technology transfer problem depending on the cost gap as a similar structure. Their technology transfer corresponds to the location choice here.

equilibrium is almost always uniquely determined. This is in contrast to the core-periphery model in the NEG, in which the symmetry-breaking and sustain points are different and hence the multiple equilibria arise.[14] In our model these two points coincide.

5 Welfare

Social welfare is another important issue to consider. Is location equilibrium desirable or not? We define the social welfare function as the sum of consumer surplus (CS) and producer surplus (PS), which gives the social surplus ($SS = CS + PS$). From the demand function in (2), the local consumer surplus derived at z, $cs(z)$, is given by

$$cs(z) = \frac{1}{2}\left[a - P(z)\right] Q(z) = \frac{b}{2} Q(z)^2, \tag{11}$$

where $P(z)$ and $Q(z)$ are the price and the total supply amount at z, respectively. Then, we have $CS = \int_0^1 cs(x)dx + n\left[cs(x_1) + cs(x_2)\right]$. PS is the sum of profits given by $PS = \Pi_1 + \Pi_2$, where Π_i is defined in (5). We analyze the first-best situation, in which the social planner is able to control both the supply amount and the location of each firm, and the second-best situation, in which the planner is only able to control the location of each firm.

5.1 First-best situation

In the first-best, it is clear that the planner must set the supply amount at each market such that the price is equal to the transport cost (marginal-cost pricing). Hence, once the locations of both firms are given, each market should be served by the nearer firm at the price of the unit transport cost. Then, total transport costs in the whole economy must be minimized for the first-best situation. Clearly, this will be achieved by full dispersion.

[14] In the literature, the symmetric structure, where two regions have the same share of mobile workers, breaks down when transport costs are less than the symmetry-breaking point. The core-periphery structure, where the core region contains all of the mobile workers and the peripheral region only has immobile workers (farmers), is sustained as long as the transport costs are less than sustain point. The multiple equilibria emerge when the symmetry-breaking point is less than the sustain point (Krugman, 1991; Fujita *et al.*, 1999).

Proposition 2 *In the first-best situation, two firms are located such that the distance between them is* $1/2$ *(full dispersion), and each market is served by the nearer firm in which the supply amount is ordered such that the price is equal to the transport cost there.*

5.2 Second-best situation

What if the social planner were able to control only locations? In this case, the supply amount for each market would be determined as the Cournot equilibrium, as described in Section 3. The first finding is as follows.

Lemma 1 *Let the supply amount for each consumer be given by Cournot competition under a given location. Then, consumer surplus is maximized when two firms are located at the same point (full agglomeration).*

Proof. See Appendix B. ∎

The lemma states that the consumers as a whole prefer uneven distribution of firms to dispersed distribution although the total transport costs are not minimized. The intuitive process behind this is as follows. From (11), a consumer surplus for each consumer is proportional to the square of the consumption. Suppose that there are two consumers and two units of the good. Then, if we assign one unit for each consumer, $CS \propto 1^2 + 1^2 = 2$. Meanwhile, if we give two units to only one consumer, we have $CS \propto 2^2 + 0^2 = 4$. Therefore, consumer surplus is greater by uneven allocations of the good.

In location equilibrium, neither firm cares about consumer surplus. Thus, we can predict the *excess dispersion* in equilibrium. The formal result is as follows.

Proposition 3 *Suppose that the social planner can only control location, and the supply amount is determined by Cournot competition given the location pair. Let*

$$TT(n) = \begin{cases} 1792n/3(22n+7)^2 & \text{when } 0 < n \leq 7/66 \\ 96n/\left(66n+7\right) & \text{when } 7/66 < n \end{cases}.$$

The second-best situation is achieved as follows:

(i) Full agglomeration: Two firms agglomerate at a point if $\tau \leq TT(n)$.

(ii) Full dispersion: Two firms separate from each other such that the distance between them is equal to $1/2$ *if* $\tau \geq \max\{8/11, TT(n)\}$.

(iii) Partial dispersion: Two firms separate from each other such that the distance between them is less than $1/2$ *if* $TT(n) \leq \tau < 8/11$ *under* $0 < n < 7/66$. *The distance is given by*

$$\left(22n + 7 + \sqrt{(22n+7)^2 - 448(n/\tau)}\right)/2.$$

Proof. See Appendix C. ■

[Insert Figure 3 here.]

Figure 3 shows the configuration of the second-best situation. A comparison between Figures 1 and 3 shows excess dispersion in equilibrium. For instance, when $n = 1$ and $\tau = 0.8$, full dispersion is achieved in equilibrium, whereas full agglomeration is the second-best situation. This is because consumers benefit by agglomerated locations (Lemma 1).

5.3 On residential choice

Thus far, we have neglected the issue of endogenous residential choice. One might think it is natural that consumers endogenously choose their locations to maximize their consumer surplus. From (11), we readily find that the consumer surplus is maximized at the location that minimizes the sum of the transport costs to both firms. Under the linear transport cost, the sum is constant for any location between the two firms, due to the constant sum of the distances. This implies that each mobile consumer has no incentive to move away from the firms' locations; therefore, our arbitrary assumption is not especially harmful.

The above discussion, however, would not hold if the transport cost were convex. Suppose that the transport cost is quadratic: $T(x_i, z) = \min\{t|x_i - z|^2, t(1 - |x_i - z|)^2\}$, $x_1 = 0$, and $0 \leq x_2 \leq 1/2$. Then, the consumer surplus is maximized at $z = x_2/2 \in (0, x_2)$. Therefore, our guess is that full agglomeration should be robust, while dispersion could not be robust if we incorporate the endogenous residential-choice problem.

6 Extensions of our model

6.1 Asymmetry

The numbers of mobile consumers could be different between firms. We first consider the case where only one firm (firm 1) attracts mobile consumers. That is, our profit function (5) is rewritten as $\Pi_i^*(x_i) = \int_0^1 \pi_i^*(z)dz + n\pi_i^*(x_1)$. In this case, firm 2 has a strong incentive to approach firm 1, whereas firm 1 has no incentive to approach firm 2. These asymmetric incentives lead to the following outcome.

Proposition 4 *When only one firm attracts mobile consumers, no location equilibrium exists in pure strategies.*

Proof. See Appendix D. ■

As a general case, firms 1 and 2 attract n_1 and n_2 mobile consumers, respectively ($n_1 > n_2 > 0$). Although this difference may cause the nonexistence of equilibrium in some parameter sets as Proposition 4 has shown, thanks to positive mobile consumers at both firms, we can show a similar result with the symmetric case described in the previous section. That is, firms tend to agglomerate when the transport cost is sufficiently low, whereas they separate when the cost is high enough.[15]

6.2 m-firm oligopoly

What if there are more than two firms? Recall that full agglomeration never occurs in equilibrium under $m(\geq 2)$-firm oligopoly in the circular city without mobile consumers (Matsushima, 2001; Shimizu and Matsumura, 2003; Matsumura and Matsushima; forthcoming). Hence, an interesting

[15] The detailed analysis is available from the author upon request.

question is whether full agglomeration is an equilibrium or not. Let us focus on a symmetric case where all firms attract n mobile consumers. In that case, we have the following answer.

Proposition 5 *Suppose that there are m firms and each firm attracts n mobile consumers. In that case, full agglomeration is an equilibrium when the transport cost is sufficiently low.*

Proof. See Appendix E. ■

With mobile consumers, full agglomeration can be achieved, irrespective of the number of firms. This implies that agglomerative forces become too strong under low transport costs. Note that this is not a unique outcome.[16]

6.3 The linear city

What if the city is linear? Without mobile consumers, central agglomeration is achieved in the linear city under spatial Cournot competition (Hamilton et al., 1989; Anderson and Neven, 1991). In the linear city, the center has a location advantage in the sense that total transport costs to serve all consumers are saved. Thus, one of the most interesting questions is whether (full) dispersion can be an equilibrium. Let the line segment $[0, 1]$ denote the city, and let the other settings be unchanged in the circular model.[17] Further, we analyze the duopoly in which each firm attracts the same number (n) of mobile consumers. Then, we have the following result.

Proposition 6 *Full dispersion (a firm is located at 0 and the other firm is located at 1) is an equilibrium if and only if* $\max\{6n/(15n+2), (n+2)/(5n+1)\} \leq \tau < 1/2$ *and* $n > 1$.

Proof. See Appendix F. ■

With mobile consumers, we have the dispersed equilibrium even in the linear city under spatial Cournot competition. As our circular city has shown, mobile consumers work as an agglomeration force under lower transport costs. Hence, we can even see full agglomeration at an edge of the linear city instead of the center under sufficiently low transport costs.

[16] See Matsushima (2001), Shimizu and Matsumura (2003), Matsumura and Matsushima (forthcoming) for a variety of equilibria without mobile consumers.

[17] The threshold for full market coverage is revised as $\tau < 1/2$.

7 Concluding remarks

We have analyzed how location is affected when some consumers are attracted to firms' locations after the firms make their location decisions (mobile consumers, variable distribution of consumers). To do so, new kinds of agglomeration and dispersion forces have emerged: When a firm is located near a rival, the profit increases due to the lower transport cost to the mobile consumers at the rival (agglomeration forces). Meanwhile, the firm should be punished by the approaching rival in reverse (dispersion force). Our analysis has shown that the agglomeration forces become dominant when the transport cost is low or the mass of mobile consumers is large or both.

Without mobile consumers, previous studies have shown that the circular city leads to the dispersed location equilibria (Pal, 1998; Matsushima, 2001; Shimizu and Matsumura, 2003), whereas the linear city exhibits the agglomerated location equilibria (Hamilton *et al.*, 1989; Anderson and Neven, 1991). In other words, the spatial form matters, not the parameter values such as transport costs. This is in sharp contrast with our model.

In our future research, we shall continue our work on endogenous residential choice, full analyses of multiple firms and the linear city, general forms of transport cost function, or other spatial motions of consumers.

Appendix

A. Proof of Proposition 1

Proof. Without loss of generality, let $x_2 = 0$ and $0 \leq x_1 = x \leq 1/2$. We will compute the best response of firm 1 against $x_2 = 0$, and let $x_1^* = \arg\max \Pi_1^*(x)$ denote the best response. Then, we have

$$\Pi_1^*(x) = a^2 \left[12\left(2n+1\right) - 6\tau\left(4nx+1\right) + \tau^2\left(1 + 12\left(5n+2\right)x^2 - 32x^3\right)\right]/108b,$$

and $\partial\Pi_1^*(x)/\partial x = 2a^2\tau^2 g_A(x)/9b$, where

$$g_A(x) = -4x^2 + (5n+2)\,x - n/\tau. \tag{A1}$$

Note that $\Pi_1^*(x)$ is cubic and the coefficient of x^3 in $\Pi_1^*(x)$ is negative. To investigate the behavior of $\Pi_1^*(x)$, we focus on $g_A(x)$. Due to $g_A(0) < 0$, $\Pi_1^*(x)$ is locally maximized at $x = 0$. From (A1), let the discriminant of $g_A(x) = 0$ be $d^A = (5n+2)^2 - 16n/\tau$. First, when $d^A \leq 0 \Leftrightarrow \tau \leq 16n/(5n+2)^2$, it is clear that $x_1^* = 0$. Second, when $d^A > 0 \Leftrightarrow 16n/(5n+2)^2 < \tau$, $g_A(x) = 0$ yields two local extremizers of $\Pi_1^*(x)$ as $\check{x}^A = \left(5n+2-\sqrt{d^A}\right)/8$ and $\hat{x}^A = \left(5n+2+\sqrt{d^A}\right)/8$ $(0 < \check{x}^A < \hat{x}^A)$. Here, due to the negative coefficient of x^3 in $\Pi_1^*(x)$, $\hat{x}^A$ is the local maximizer. Then, we can classify the solution of the maximization problem into three cases:

(i) full agglomeration: $x_1^* = 0 \Leftrightarrow \hat{x}^A \geq 1/2$ and $\Pi_1^*(0) \geq \Pi_1^*(1/2)$; or $0 < \hat{x}^A < 1/2$ and $\Pi_1^*(0) \geq \Pi_1^*(\hat{x}^A)$.

(ii) full dispersion: $x_1^* = 1/2 \Leftrightarrow \hat{x}^A \geq 1/2$ and $\Pi_1^*(1/2) \geq \Pi_1^*(0)$.

(iii) partial agglomeration: $x_1^* = \hat{x}^A(< 1/2) \Leftrightarrow \hat{x}^A < 1/2$ and $\Pi_1^*(\hat{x}^A) \geq \Pi_1^*(0)$.

As we have discussed the logic in Section 4, due to the symmetry, the best response of firm 2 must be $x_2^* = 0$ when firm 1 is located at x_1^* that is derived above in each case. By reducing and arranging the above conditions, we have the result. ■

B. Proof of Lemma 1

Proof. Without loss of generality, we assume $0 \leq x_1 = x \leq 1/2$ and $x_2 = 0$. Substituting the quantity equilibrium obtained in (6), (7) and (8) into the consumer surplus, we have $CS = a^2 g_B(x)/54b$, where $g_B(x) = 4\tau^2x^3 + 3(2n-1)\tau^2x^2 - 24n\tau x + \tau^2 - 6\tau + 12(2n+1)$ and $\partial g_B(x)/\partial x = 6\left[2\tau^2x^2 + (2n-1)\tau^2 x - 4n\tau\right]$. $\partial g_B(x)/\partial x = 0$ yields two local extremizers as $\check{x}^B = \left(1-2n-\sqrt{d^B}\right)/4$ and $\hat{x}^B = \left(1-2n+\sqrt{d^B}\right)/4$, where $d^B = (1-2n)^2 + 32(n/\tau) > 0$. Under $n > 0$ and $0 < \tau < 1$, we readily have $\check{x}^B < 0$ and $1/2 < \hat{x}^B$. Because of the positive coefficient of x^3 in $g_B(x)$, $g_B(x)$ is monotonically decreasing in $x \in [0, 1/2]$. Therefore, full agglomeration maximizes consumer surplus. ■

C. Proof of Proposition 3

Proof. Without loss of generality, $x_2 = 0$ and $0 \leq x_1 = x \leq 1/2$. Then, the social surplus is rewritten by $SS(x) = a^2 g_C(x)/54b$, where

$$SS(x) = a^2 \left[-28\tau^2 x^3 + 3(22n+7)\tau^2 x^2 - 48n\tau x + 2\tau^2 - 12\tau + 24(2n+1)\right]/54b,$$

and $\partial SS(x)/\partial x = a^2\tau^2 g_C(x)/9b$, where

$$g_C(x) = -14x^2 + (22n+7)x - 8\left(n/\tau\right). \tag{C1}$$

Note that $SS(x)$ is cubic and the coefficient of x^3 in $SS(x)$ is negative. Due to $g_C(0) < 0$, $SS(x)$ is locally maximized at $x = 0$. From (C1), let the discriminant of $g_C(x) = 0$ be $d^C = (22n+7)^2 - 448n/\tau$. First, when $d^C \leq 0 \Leftrightarrow \tau \leq 448n/(22n+7)^2$, it is clear that $\arg\max SS(x) = 0$. Second, when $d^C > 0 \Leftrightarrow 448n/(22n+7)^2 < \tau$, $g_C(x) = 0$ yields two local extremizers of $SS(x)$ as $\check{x}^C = \left(22n+7-\sqrt{d^C}\right)/28$ and $\hat{x}^C = \left(22n+7+\sqrt{d^C}\right)/28$ ($0 < \check{x}^C < \hat{x}^C$). Due to the negative coefficient of x^3 in $SS(x)$, $\hat{x}^C$ is the local maximizer. Then, we analyze three cases like Appendix A.

(i) full agglomeration: $\arg\max SS(x) = 0 \Leftrightarrow \hat{x}^C \geq 1/2$ and $SS(0) \geq SS(1/2)$; or $0 < \hat{x}^C < 1/2$ and $\Pi_1^*(0) \geq \Pi_1^*(\hat{x}^C)$.

(ii) full dispersion: $\arg\max SS(x) = 1/2 \Leftrightarrow \hat{x}^C \geq 1/2$ and $SS(1/2) \geq SS(0)$.

(iii) partial agglomeration: $\arg\max SS(x) = \hat{x}^C(< 1/2) \Leftrightarrow \hat{x}^C < 1/2$ and $SS(\hat{x}^C) \geq \Pi_1^*(0)$.

By reducing and arranging the above conditions, we have the result. ■

D. Proof of Proposition 4

Proof. Without loss of generality, firm 1 attracts n mobile consumers whereas firm 2 attracts no mobile consumers. Then, the density function of mobile consumers at z in (1) is rewritten by $f(z) = 0$ if $z \neq x_1$; $f(z) = n$ if $z = x_1$. And the profit functions are also rewritten adequately. Like Appendix A, we seek the best response of firm 1. Without loss of generality, $x_2 = 0$ and $0 \leq x_1 = x \leq 1/2$. Then, we obtain

$$\Pi_1^*(x) = a^2 \left[12(1+n) + 6\tau(4nx-1) + \tau^2\left(1 + 12(n+2)x_1^2 - 32x_1^3\right)\right]/108b$$

and $\partial\Pi_1^*(x)/\partial x = 2a^2\tau^2 g_D(x)/9b$, where

$$g_D(x) = -4x_1^2 + (n+2)x + n/\tau. \tag{D1}$$

Note that $\Pi_1^*(x)$ is cubic and the coefficient of x_1^3 is negative. From (D1), $g_D(x) = 0$ yields two local extremizers of $\Pi_1^*(x)$ as $\check{x}^D = \left(n + 2 - \sqrt{d^D}\right)/8$ and $\hat{x}^D = \left(n + 2 + \sqrt{d^D}\right)$, where $d^D = (n+2)^2 + 16(n/\tau) > 0$. Because $\check{x}_1^D < 0 < 1/2 < \hat{x}_1^D$, $\Pi_1^*(x)$ is increasing in $x \in [0, 1/2]$. Hence, the best response of firm 1 is always 1/2.

Next, we analyze the reaction of firm 2. Let $x_1 = 0$ and $0 \leq x_2 = x \leq 1/2$. Then, we have

$$\left.\frac{\partial\Pi_2^*(x)}{\partial x}\right|_{x=1/2} = \frac{4n\tau}{9b}(\tau - 1) < 0.$$

Therefore, firm 2 never chooses a location such that the distance between two firms is 1/2. Thus, there is no equilibrium in pure strategies. ■

E. Proof of Proposition 5

Proof. (We apply the same notations as in the duopoly case.) The total number of the firms is m, and let $j = \{1, 2, \ldots, m\}$ and x_j denote the firm's index and the location of firm j, respectively. Inverse demand function at z is revised as $P(z) = a - b\sum_j q_j(z)$, where $q_j(z)$ is supply amount of firm j at z. The total profit of firm i is given by $\Pi_i = \int_0^1 \pi_i(z)dz + n\pi_i(x_i) + n\sum_{j\neq i}\pi_i(x_j)$, where $\pi_i(z) = q_i(z)\left[P(z) - T(x_i, z)\right]$ is the local profit for firm i at z. Then, the similar calculations yield equilibrium supply amount at z as follows:

$$q_i^*(z) = \frac{a - mT(x_i, z) + \sum_{j\neq i} T(x_j, z)}{(m+1)\,b}.$$

Here, we require $\tau < 2/m$ for each market to be served by all firms, irrespective of the locations of the firms. Substituting the above equilibrium quantity into the profit functions, we can rewrite the local and the total profits for firm i as $\pi_i^*(z) = b\left[q_i^*(z)\right]^2$ and $\Pi_i^*(x_i) = \int_0^1 \pi_i^*(z)dz + n\pi_i^*(x_i) + n\sum_{j\neq i}\pi_i^*(x_j)$, respectively. Then, we check the relocation incentive for a firm when all firms agglomerate at a location. Without loss of generality, all firms agglomerate at 0 and we analyze the optimal location for firm 1 with $0 \leq x_1 = x \leq 1/2$. Then, we have the profit function of firm 1 as follows:

$$\begin{aligned} \frac{12b\,(1+m)^2}{a^2} \times \Pi_1^*(x) &= 12\,(1+mn) - 6\tau\left[1 + 4(m-1)^2 nx\right] \\ &\quad + \tau^2\left[1 + 12\,(m-1)\left(m^2 n + mn + m - n\right)x^2 - 16m\,(m-1)\,x^3\right]. \end{aligned}$$

Note that the profit function is cubic and the coefficient of x_1^3 is negative, and $\partial\Pi_1^*(x)/\partial x|_{x=0} < 0$. Let d^E be a discriminant of $\partial\Pi_1^*(x_1)/\partial x_1 = 0$. If $d^E \leq 0$, then the optimal location $x_1^* = 0$. When $d^E > 0$, $\partial\Pi_1^*(x)/\partial x = 0$ yields the local minimizer and the local maximizer as $\check{x}^E$ and $\hat{x}^E$, respectively ($0 < \check{x}^E < \hat{x}^E$). For $x = 0$ to be the optimal location, we require $\Pi_1^*(0) > \Pi_1^*(1/2)$ if $\hat{x}^E \geq 1/2$, or $\Pi_1^*(0) > \Pi_1^*(\hat{x}^E)$ if $\hat{x}^E < 1/2$. By reducing these conditions, we have

$$0 < \tau < T(m,n) \implies \arg\max \Pi_1^*(x) = 0,$$

where

$$T(m,n) = \begin{cases} 32(m-1)mn/3(m^2n+mn+m-n)^2 & \text{when } 0 < n \leq m/3(m^2+m-1) \\ 12(m-1)n/\left(3m^2n+3mn+m-3n\right) & \text{when } m/3(m^2+m-1) < n \end{cases} .$$

For any $m \geq 3$ and for any $n > 0$, $T(m,n) > 0$. This implies that if the transport cost is sufficiently low such that $\tau < T(m,n)$ and $\tau < 2/m$, full agglomeration is a location equilibrium. ■

F. Proof of Proposition 6

Proof. Without loss of generality, let firm 2 be located at 1 ($x_2 = 1$). The equilibrium supply amount at z for firm 1 is given by $q_1^*(z) = \left[a - 2T(x_1,z) + T(x_2,z)\right]/3b$. We require $\tau < 1/2$ for each market to be served by both firms, irrespective of their locations. The total profit for firm 1 is given by $\Pi_1^*(x_1) = a^2 g_F(x_1)/27b$, where

$$g_F(x_1) = 4\tau^2x_1^3 + 3(5n\tau-4)x_1^2 - 6\tau\left[(5\tau-1)n+\tau-2\right]x_1 + 3\left[(5\tau^2-2\tau+2)n+\tau^2-\tau+1\right].$$

Note that $g_F(x_1)$ is cubic and the coefficient of x_1^3 is positive. If the discriminant of $\partial g_F(x_1)/\partial x_1 = 0$ is zero or negative, then $g_F(x_1)$ is non-decreasing, i.e., $\arg\max g_F(x_1) \neq 0$. Suppose that the discriminant is positive, and let $\check{x}_1^F$ and $\hat{x}_1^F$ ($\check{x}_1^F \leq \hat{x}_1^F$) denote the local minimizer and the local maximizer of $g_F(x_1)$. For $\arg\max g_F(x_1) = 0$ to hold, we require that $\Pi_1^*(0) \geq \Pi_1^*(1)$ and $\check{x}_1^F \leq 0$. Tedious calculations yield $\Pi_1^*(0) \geq \Pi_1^*(1) \iff \tau \geq 6n/(15n+2)$ and $\check{x}_1^F \leq 0 \iff \tau \geq (n+2)/(5n+1)$. When $n \leq 1$, these conditions violate the full-coverage condition $\tau < 1/2$. In other words, full dispersion is an equilibrium for any τ satisfying the above conditions only when $n > 1$. ■

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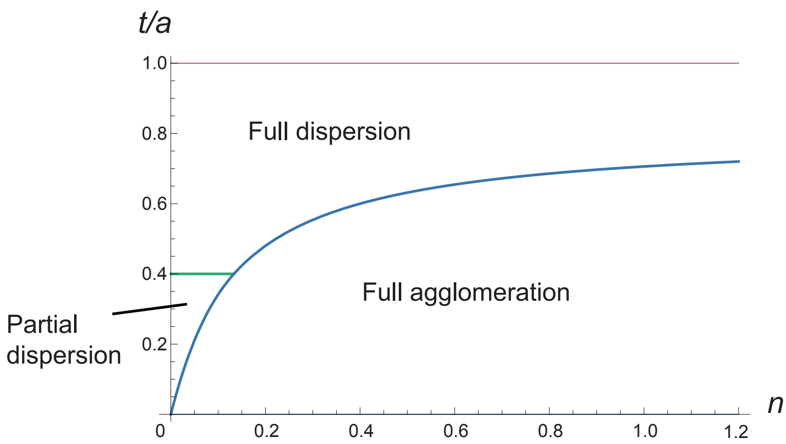

Figure 1: The classification of the location equilibrium

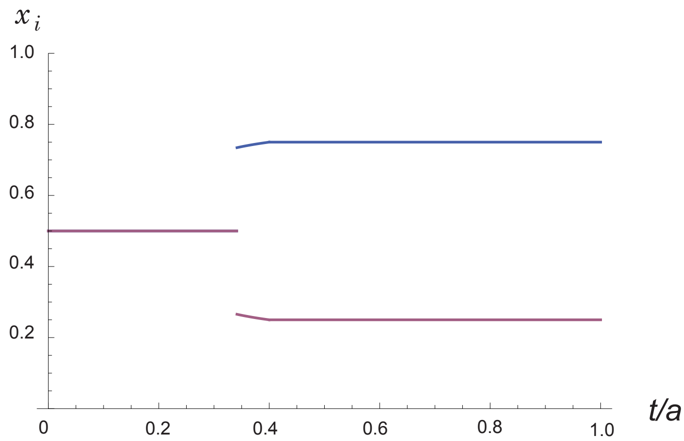

Figure 2: The pitchfork diagram (*n=1/10*)

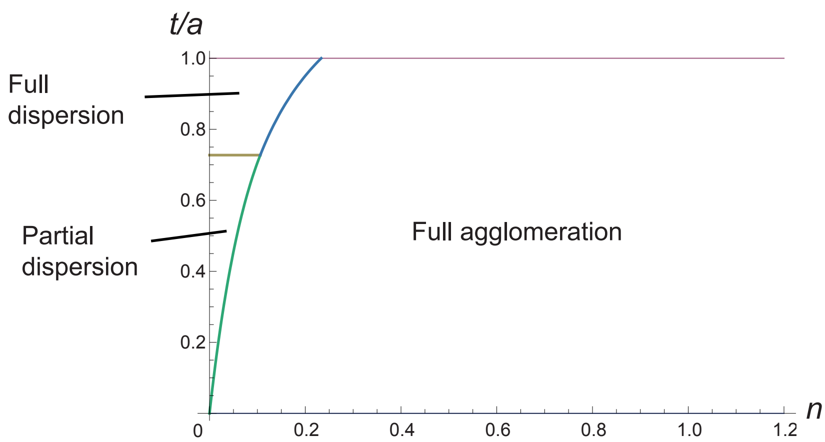

Figure 3: The classification of the second-best situation

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