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Spatial competition with home bias

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Abstract

This paper analyzes the location equilibrium for duopolistic firms in which consumers prefer a good produced by a nearer firm when the prices of both goods are equal (home bias, or vertical differentiation). We consider a linear space, and the firms choose their locations prior to quantity (price) competition. The nearer a firm is to a market, the higher the quality of its good for the market becomes. As a result, we compute numerically to show that there exists a symmetric dispersed location equilibrium. When transport costs are decreasing, the firms approach toward the center each other in quantity competition. However, in price competition, the result is reversed: The firms separate each other. This contrastive result is because expanding the market share takes precedence over avoiding the competition in quantity competition. Meanwhile, in price competition, it more matters to avoid keen price competition.

JEL Classification: L13, R12, R30.

Keywords: home bias, location, vertical product differentiation, mismatch cost

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1 Introduction

This paper develops a spatial competition model à la Hotelling (1929) with home bias in which consumers prefer a good produced by a nearer firm when the prices of both goods are equal. In other words, we deal with a model of vertical differentiation (henceforth VPD) endogenously determined by locations. Hotelling (1929) analyzed a location-then-price competition model with duopolistic firms selling a homogeneous commodity to consumers uniformly distributed over a line segment; then, he showed the firms agglomerate in the center. This result is termed *principle of minimum differentiation*. D'Aspremont, Gabszewicz and Thisse (1979) have corrected this principle. They adopted a quadratic transport cost function, and showed that the two firms mutually locate themselves apart from each other at the edges of the market. *Principle of maximum differentiation* arises in this location-then-price competition model. This shows a strong tendency for firms to avoid keen price competition by locational dispersion.

As another trend for spatial competition, Hamilton, Thisse and Weskamp (1989) and Anderson and Neven (1991) incorporated a quantity competition à la Cournot into spatial competition models. They consider that each point over the line segment has a Cournot market, and firms engage in location-then-quantity competition. Consequently, both studies showed *principle of minimum differentiation*, where all firms agglomerate in the center. Hence, the results of price competition and quantity one are contrastive. This paper analyzes both types of competition in order to better understand the differences.

Unlike those studies with a homogeneous good, this paper deals with a model of a differentiated product. When it comes to a spatial competition with product differentiation, De Fraja and Norman (1993) analyzed a spatial model with a *horizontally* differentiated good. In their model, given a differentiation parameter set representing elasticity of substitution, duopolistic firms engage in location-then-price competition. As a result, under a sufficient differentiation, the firms agglomerate in the center. We adopt a VPD model instead of a horizontal one.

It is *location determines product quality* that is the most special feature in our model.[1] Quality of a good produced by a nearer firm is assumed to be higher than that produced by a farther firm for each consumer. Note that because the consumers are distributed over a linear city, the evaluation for the quality of a good varies in locations of the markets. In the real world, we can imagine the following examples. First, we often prefer a product from the home country to that from foreign countries; thus, we would like to buy one from the home rather than the others even if the prices of those goods are all equal. [2] That means quality depends on location. Second, we can regard the space as a characteristic space rather than a geographical one. In Japanese cities that have small land and concentrated population, people there prefer a smaller car that spares space and is easily driven in a narrow road. On the other hand, in the US cities that are relatively wider, consumers may prefer a bigger car.

Our main result is that the duopolistic firms locate apart each other to some extent. It is natural that they never agglomerate. If so, they do produce the same product (homogeneous product) to be involved by a severe competition. As Motta (1993) has shown, when the firms can choose the qualities of their goods, they offer the different qualities each other to avoid such a competition. Further, the dispersion in location is more under Bertrand than under Cournot, which is also consistent with Motta (1993). This is because Bertrand competition is more severe, then the firms have a stronger incentive to locate apart in order to avoid competition.

When transport costs are decreasing, the firms approach toward the center each other when they engage in quantity competition. However, in price competition, this result is reversed: The firms separate each other. This contrastive result is because quantity competition is less competitive. In other words, expanding the market share takes precedence over avoiding the competition in quantity competition. Meanwhile, in price competition, it more matters to avoid keen price competition

[1] In most models of VPD, quality is enhanced by (R&D) investment with cost that is a direct strategic variable. However, in this paper, in order to concentrate on location factors, we omit such a direct choice of quality.

[2] The strict definition of (higher) quality in VPD is: when all consumers always buy a good rather than another good if the prices of them are equal, quality of the good that is bought is higher than that of another.

caused by low transport costs.

The remainder of the paper is organized as follows. In Section 2, the two-stage game is presented and the games (price and quantity competition) at the second stage are also analyzed. In Section 3, we evaluate the equilibrium and computes some economic values: prices, profits, and consumer surplus in the equilibrium for a parameter set. Section 4 summarizes the results.

2 The model

We consider a linear city expressed by the line segment, $L = [0, 1]$. There are two competing firms indexed by i $(i = 1, 2)$. Let the firms' locations be $x_1 \equiv x$ and $x_2 \equiv 1 - y$, respectively $(x_i \in L)$; and each firm produces a differentiated good (good i). Each point in the city has a set of consumers that have different tastes for quality and buy either of the good inelastically or do not buy it at all. Consumers at z have the same utility function (and zero utility if they do not buy the good). When they buy good i, they get the utility as

$$U(z) = u_i(z)v - p_i(z),$$

where $u_i(z)$ and $p_i(z)$ are the quality and the price of good i at z respectively; and $v \in [\underline{v}, \overline{v}]$, being uniformly distributed with unit density, is a taste parameter.[3] The quality of a good in a market becomes higher when the producer locates nearer to the market. Then, we assume

$$u_i(z) = a - t\left|z - x_i\right|,$$

[3] This setting is based on a familiar VPD model by Shaked and Sutton (1982).

where a ($> t > 0$) is reserved quality and t is a mismatch cost parameter.[4] Let $m \equiv (x_1 + x_2)/2 = (1 + x - y)/2$. Then, for the consumers over $[0, m)$ (resp. $(m, 1]$), the quality of good 1 (resp. good 2) is higher.

We consider a two-stage location-price game and a two-stage location-quantity one. The structure of the games are based on Motta (1993), where two firms simultaneously choose their qualities at the first stage; then they compete in quantity (or price) at the second stage. He analyzed a non-spatial, single market where the firms directly choose their qualities with costs. Meanwhile, our firms only can choose their qualities through location choices. Further, we consider a continuum mass of markets in the linear city. The other structure is the same as in Motta (1993).

2.1 Bertrand competition

2.1.1 Price equilibrium

First, we analyze Bertrand competition at the second stage.[5] We consider a market at $z \in [0, m)$, where the quality of good 1 is higher.[6] The consumer indifferent between buying good 1 or good 2 has a taste parameter v_{12} such that $v_{12} = [p_1(z) - p_2(z)] / [u_1(z) - u_2(z)]$. The consumer indifferent between buying good 2 and not buying at all has a taste parameter $v_{\phi 2} = p_2(z)/u_2(z)$. Following Motta (1993), we assume that the market is not covered; that is, $\underline{v} < v_{\phi 2}$ for any z. Thus, we have each firm's demand $q_i(z)$ as follows:

$$q_1(z) = \overline{v} - [p_1(z) - p_2(z)] / [u_1(z) - u_2(z)], \tag{1}$$

$$q_2(z) = [p_1(z) - p_2(z)] / [u_1(z) - u_2(z)] - p_2(z)/u_2(z). \tag{2}$$

[4] The assumption about a ensures a positive quality value and positive demands for both firms for all locations. We exclude direct quality competitions like R&D, and the generic product quality (a) is given and the same for both firms.

[5] Here, we exclude the case of $x_1 = x_2$. In this agglomeration case, there is no differentiation for the entire market, and the prices becomes marginal cost, 0. We can readily show that this is not an equilibrium.

[6] At $z = m$, because there is no differentiation, the prices and the profits clealy become zero.

Firms choose prices to maximize their local profits $\pi_i(z) = p_i(z)q_i(z)$ for any quality pair $(u_1(z), u_2(z))$ specified by their locations. From the first-order conditions, we have the following equilibrium prices:

$$p_1(z) = 2\overline{v}u_1(z)\left[u_1(z) - u_2(z)\right] / \left[4u_1(z) - u_2(z)\right], \tag{3}$$

$$p_2(z) = \overline{v}u_2(z)\left[u_1(z) - u_2(z)\right] / \left[4u_1(z) - u_2(z)\right]. \tag{4}$$

The corresponding local profits are given by:

$$\pi_1(z) = 4\left[u_1(z) - u_2(z)\right]\left\{\overline{v}u_1(z)/\left[4u_1(z) - u_2(z)\right]\right\}^2 \quad \text{if} \quad z \in [0, m) \ , \tag{5}$$

$$\pi_2(z) = u_1(z)u_2(z)\left[u_1(z) - u_2(z)\right]\left\{\overline{v}/\left[4u_1(z) - u_2(z)\right]\right\}^2 \quad \text{if} \quad z \in [0, m) \ \cdot \tag{6}$$

We proceed the remaining markets over $(m, 1]$, where the quality of good 2 is higher. Due to the symmetry, exchanging the firms' indexes in equations (1)-(6) generates the equilibrium values. The local profits are:

$$\pi_1(z) = u_1(z)u_2(z)\left[u_2(z) - u_1(z)\right]\left\{\overline{v}/\left[4u_2(z) - u_1(z)\right]\right\}^2 \quad \text{if} \quad z \in (m, 1], \tag{7}$$

$$\pi_2(z) = 4\left[u_1(z) - u_2(z)\right]\left\{\overline{v}u_1(z)/\left[4u_1(z) - u_2(z)\right]\right\}^2 \quad \text{if} \quad z \in (m, 1]. \tag{8}$$

Hence, we can calculate the total profits in Bertrand competition as a function of the location pair:

$$\Pi_i^B(x, y) = \int_0^1 \pi_i(z)dz, \tag{9}$$

where $\pi_i(z)$ is given by (5)-(8).

2.1.2 Location equilibrium

Here, we proceed the location competition in the first stage. We define a location equilibrium as a location pair in which no firm can earn a greater profit by relocation when the other firm's location is fixed. Hereafter, let $\tau \equiv t/a$ $(0 < \tau < 1)$ as the mismatch cost measure. Unfortunately, due to the nonlinearity and complexity of equations, we will compute equilibria approximately by Newton method. However, some properties of equilibria can be shown analytically. First, we can show that a candidate of corner solutions, $x = 0$ or $y = 0$ cannot be an equilibrium. Second, we readily have the fact that $\partial \Pi_1^B(x, y)/\partial x$ and $\partial \Pi_2^B(x, y)/\partial y$ are symmetric with regard to $x = y$. Hence, the equilibria are, if any, to be symmetric with regard to the center ($x = y$). At last, we have $\Pi_i^B(1/2, 1/2) = 0$ for $i = 1, 2$, and

$$\left.\frac{\partial \Pi_1^B}{\partial x}(x, y)\right|_{x=1/2, y=1/2} = \left.\frac{\partial \Pi_2^B}{\partial y}(x, y)\right|_{x=1/2, y=1/2} = -\frac{5}{18} a\tau\overline{v}^2 < 0.$$

Hence, we have the following fact.

Remark 1 *The central agglomeration cannot be an equilibrium.*

This is straightforward because the central agglomeration leads to zero profits for the firms. They can earn positive profits by locational dispersion. Hence, we will focus on symmetric, inner solutions. Note that $\Pi_1^B(x, y)$ and $\Pi_2^B(x, y)$ are identical when $x = y$. We have

$$\left.\frac{\partial \Pi_i^B}{\partial x}(x, y)\right|_{y=x} = \frac{-a\overline{v}^2 f^B}{33750\left(3 + \tau - 5\tau x\right)^2 \left(3 + \tau - 2\tau x\right)^2},$$

where

$$f^B \equiv -32500\tau\left(2x - 1\right)\left[9 + 3\tau\left(2 - 7x\right) + \tau^2\left(1 - 7x + 10x^2\right)\right]^2 \ln\left(3 + \tau - 5x\tau\right) + ...$$

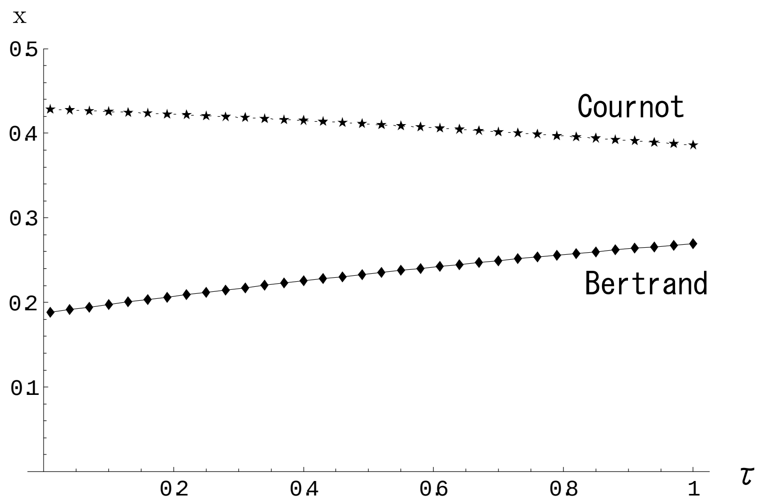

Figure 1: Location equilibrium: Quantity competition (upper line), Price competition (lower line).

Note that f^B is a function of x and τ and the location equilibria must satisfy $f^B = 0$. We proceed a computational analysis; then, we find a unique equilibrium ($x = y = x^B$) as the upward sloping connected line in Figure 1.[7]

As the limiting cases, we have the following values:

$$x^B = 0.1875 \qquad \text{when } \tau \to 0 \ (a \to \infty)$$

$$x^B = 0.269605 \qquad \text{when } \tau \to 1 \ (a \to t)$$

2.2 Cournot competition

2.2.1 Quantity equilibrium

Next, we consider Cournot competition. Consider a market at $z \in [0, m)$, where the quality of good 1 is higher. Inverting the system of demand functions given by (1) and (2), we have inverse demand

[7] We can find that these equilbria satisfy the second-order conditions.

functions as follows:

$$p_1(z) = \overline{v}u_1(z) - q_1(z)u_1(z) - q_2(z)u_2(z), \tag{10}$$

$$p_2(z) = [\overline{v} - q_1(z) - q_2(z)]\, u_2(z). \tag{11}$$

The firms choose quantities to maximize their local profits $\pi_i(z) = p_i(z)q_i(z)$, for any given quality pair $(u_1(z), u_2(z))$. From first-order conditions, we have the following equilibrium quantities:

$$q_1(z) = \overline{v}\,[2u_1(z) - u_2(z)] \,/\, [4u_1(z) - u_2(z)]\,, \tag{12}$$

$$q_2(z) = \overline{v}u_1(z)/\,[4u_1(z) - u_2(z)]\,. \tag{13}$$

The corresponding local profits are given by:

$$\pi_1(z) = u_1(z)\left\{\overline{v}\,[2u_1(z) - u_2(z)] \,/\, [4u_1(z) - u_2(z)]\right\}^2 \quad \text{if} \quad z \in [0, m]\ , \tag{14}$$

$$\pi_2(z) = u_2(z)\left\{\overline{v}u_1(z)/\,[4u_1(z) - u_2(z)]\right\}^2 \quad \text{if} \quad z \in [0, m]\ . \tag{15}$$

Note that these profits are always positive even when $u_1(z) = u_2(z)$. We proceed the remaining markets over $(m, 1]$, where the quality of good 2 is higher. Due to the symmetry, exchanging the firms' indexes in equations (10)-(15) generates the equilibrium values. The local profits are:

$$\pi_1(z) = u_1(z)\left\{\overline{v}u_2(z)/\,[4u_2(z) - u_1(z)]\right\}^2 \quad \text{if} \quad z \in (m, 1], \tag{16}$$

$$\pi_2(z) = u_2(z)\left\{\overline{v}\,[2u_2(z) - u_1(z)] \,/\, [4u_2(z) - u_1(z)]\right\}^2 \quad \text{if} \quad z \in (m, 1]. \tag{17}$$

Hence, we can calculate the total profits in Cournot competition as a function of the location pair:

$$\Pi_i^C(x, y) = \int_0^1 \pi_i(z)dz, \tag{18}$$

where $\pi_i(z)$ is given by (14)-(17).

2.2.2 Location equilibrium

We proceed the location competition in the first stage. Again, due to the nonlinearity of equations, we have equilibria numerically. As in the Bertrand case, corner solutions cannot be an equilibrium, and $\partial\Pi_1^C(x,y)/\partial x$ and $\partial\Pi_2^C(x,y)/\partial y$ are symmetric with regard to $x=y$. Hence, the solutions are, if any, to be symmetric with regard to the center. Further, we have

$$\left.\frac{\partial\Pi_1^C}{\partial x}(x,y)\right|_{x=1/2,y=1/2}=\left.\frac{\partial\Pi_2^C}{\partial y}(x,y)\right|_{x=1/2,y=1/2}=-\frac{1}{27}a\tau\overline{v}^2<0.$$

Hence, the central agglomeration cannot be an equilibrium again. We will focus on symmetric, inner solutions henceforth. From (18), we have

$$\left.\frac{\partial\Pi_i^C}{\partial x}(x,y)\right|_{y=x}=\frac{-a\overline{v}^2f^C}{5625\left(3+\tau-5\tau x\right)^2\left(3+\tau-2\tau x\right)^2},$$

where

$$f^C\equiv 1250\tau\left(2x-1\right)\left[9+3\tau\left(2-7x\right)+\tau^2\left(1-7x+10x^2\right)\right]^2\ln\left(3+\tau-5x\tau\right)+...$$

Note that f^C is a function of x and τ. The location equilibria must satisfy $f^C=0$; then, we have a unique equilibrium ($x=y=x^C$) as in Figure 1. As the limiting cases, we have the following values:

$$\begin{array}{ll} x^C=0.428571 & \text{when } \tau\to 0\ (a\to\infty) \\ x^C=0.38613 & \text{when } \tau\to 1\ (a\to t) \end{array}.$$

3 Discussion

We can summarize the result as the following remarks.

Remark 2 *With regard to interior location equilibria, there uniquely exists the symmetric dispersed equilibrium for both price competition and quantity competition (no agglomeration). The firms are more dispersed in Bertrand competition than in Cournot competition for any mismatch costs.*

This result is straightforward and is consistent with Motta (1993). Because Bertrand competition is more intense than under Cournot competition, firms have a stronger incentive to separate from the rival firm in the former case. Especially, agglomeration in the center makes no differentiation at all and the most severe competition. That is why both firms never agglomerate.

However, the reactions to the changes of mismatch costs are different between the competitions.

Remark 3 *The less the mismatch costs are; the more the firms separate in Bertrand competition, while the nearer to the center they locate in Cournot competition.*

This result is not so straightforward. A decrease in the mismatch costs strengthens both the contrastive effects: (1) pro-competitive effect and (2) market expanding effect. The former is due to less difference in quality that leads the firms to intense price competition, which is a centrifugal effect. The latter is from the competitiveness at relatively remote markets, which is a centripetal effect. In Bertrand competition, pro-competitive effect is so dominant that the firms separate from the rival. On the other hand, in Cournot competition, the competition is not so intense that the firms rather approach each other to earn more profits from the entire market.

To confirm these intuitive explanation, we compute the prices and the total profits under $(a, \overline{v}) = (5, 5)$. Figure 2 and Figure 3 show the equilibrium prices at $z = 0.45$ and $z = 0.1$ respectively. In the figures, the superscript shows the competition type: B is Bertrand and C is Cournot. The subscript indicates the firm: H is high quality (firm 1) and L is low quality (firm 2).

These figure shows that the prices under Bertrand approach down to zero due to intense price competition by a decrease in the mismatch costs. Meanwhile, when the mismatch costs decrease, the prices for the low quality firm under Cournot go up and approach to a positive value with the

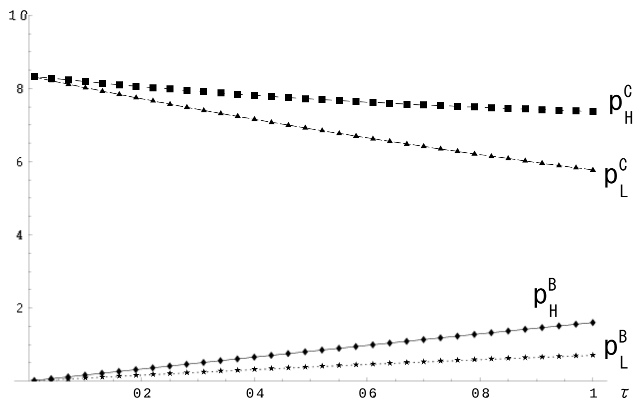

Figure 2: The equilibrium price at $z = 0.45$.

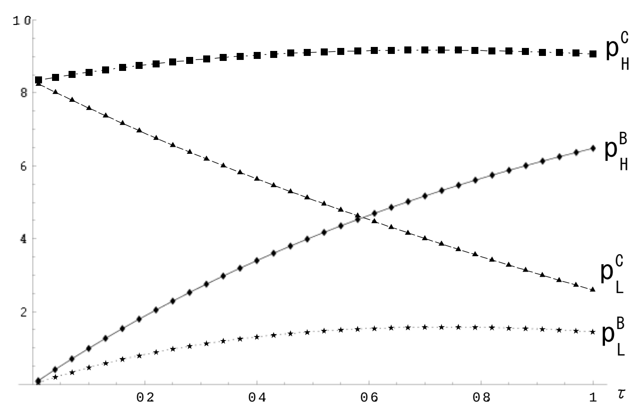

Figure 3: The equilibrium price at $z = 0.1$.

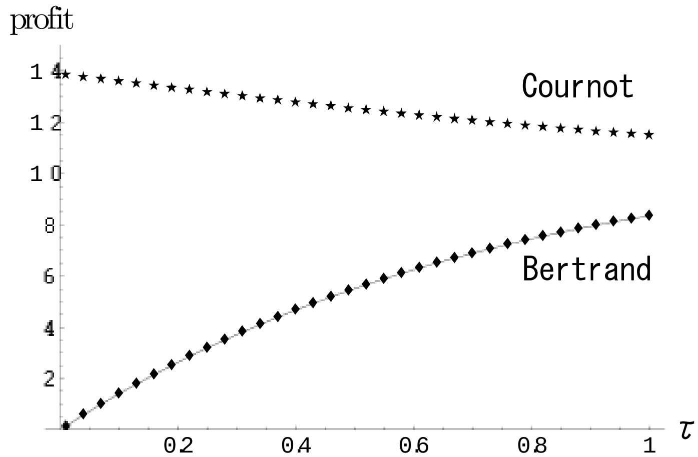

Figure 4: Profits under Bertrand and Cournot in the location equilibrium.

prices for the high quality firm. Hence, a decrease in the mismatch costs is less welcome for the firms under Bertrand, and they try to locate apart in order to avoid such a competition. On the other hand, under Cournot competition, a decrease in the mismatch costs does not lead a intense price competition; thus, the firms approach to the center to expand the market share. Figure 4 reconfirms such an explanation by computing the equilibrium total profits under Bertrand and Cournot.

4 Concluding Remarks

We have developed a new type of spatial competition model: Location determines quality. We also have considered the two types of competition: Bertrand and Cournot. One of our results is that the firms choose dispersed locations, and neither maximum nor minimum differentiation arises for both competition settings. Another result is somewhat surprising that the reactions to the mismatch costs are different between the competitive nature. When mismatch costs are decreasing, the firms approach toward the center (resp. separate) each other when they engage in quantity (resp. price) competition. This is because quantity competition is less competitive; then, expanding the market share matters more than avoiding the competition in quantity competition. Meanwhile, in price competition, it is more important to avoid keen price competition caused by low mismatch costs.

References

[1] Anderson, S.P. and A. de Palma (1988), "Spatial Price Discrimination with Heterogeneous Products," *Review of Economic Studies* 55, 573-592.

[2] Anderson, S. P. and D. J. Neven (1991), "Cournot competition yields spatial agglomeration," *International Economic Review* 32, 793-808.

[3] d'Aspremont, C., J. J. Gabszewicz, and J.-F. Thisse (1979), "On Hotelling's 'Stability In Competition'," *Econometrica* 47 (5), 1045-1050.

[4] De Fraja, G. and G. Norman (1993), "Product differentiation, pricing policy and equilibrium," *Journal of Regional Science* 33, 343-363.

[5] Hamilton, J. H., J.-F. Thisse, and A. Weskamp (1989), "Spatial discrimination, Bertrand vs. Cournot in a model of location choice," *Regional Science and Urban Economics* 19, 87-102.

[6] Hotelling, H. (1929), "Stability in competition," *Economic Journal* 39, 41-57.

[7] Motta, M. (1993), "Endogenous quality choice: price vs. quantity competition," *The Journal of Industrial Economics* 41 (2), 113-131.

[8] Shaked, A. and J. Sutton (1982), "Relaxing price competition through product differentiation," *Review of Economic Studies* 49, 3-13.

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