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Abstract

This paper analyzes a spatial Cournot competition model in a two-dimensional rectangular city. Consequently, there exists a unique spatial equilibrium such that both firms agglomerate in the center of the city with sufficiently low transport costs.

Keywords: spatial Cournot competition, two-dimensional city, central agglomeration

JEL Classification: L13; L15; R39

1 Introduction

Hotelling's (1929) seminal work showed that duopolistic firms agglomerate in the center of a one-dimensional space (a linear city). Hamilton *et al.* (1989) and Anderson and Neven (1991) developed location-then-quantity (Cournot)

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competition models rather than Bertrand ones. They then showed the agglomeration of firms in the center, while spatial Bertrand competition shows no agglomeration (see, e.g., d'Aspremont *et al.* (1979)).

This paper extends such a spatial Cournot competition model to a two-dimensional rectangular city.[1] Maldonado *et al.* (2005) have already shown that both firms agglomerate in the center when the space is a disk (a circular city). However, such a circular city is difficult to interpret when we consider the space to be a characteristic one instead of a geographical one. Further, because the ratio of the length to the breadth of our rectangle can vary, our model contains more types of spaces. That is why the analysis of a rectangular city has merit.

We consider a location-then-quantity competition game involving duopolists; then, we show the same result of central agglomeration with sufficiently low transport costs. In other words, the central agglomeration is robust in a spatial Cournot competition for a wide range of spaces.

2 The model

We consider a city expressed by a rectangle, $L = \{(x, y) \in \mathbb{R}^2 : -l_x/2 \leq x \leq l_x/2, -l_y/2 \leq y \leq l_y/2\}$, where l_x and l_y are constants ($l_x \geq l_y$) and consumers are uniformly and continuously distributed with a density of one at each location on L. The total mass of the consumers is normalized to one; thus, we can rewrite

[1] With regard to spatial Bertrand competition models in a multiple dimensional space, see, e.g., Tabuchi (1994) and Irmen and Thisse (1998).

$l_x = l$ and $l_y = 1/l$ ($l \geq 1$). There are two firms (firm 1 and firm 2) that supply a homogeneous good with zero marginal costs and engage in Cournot competition. Each consumer has the same inverse demand function as follows:

$$P = a - bQ, \quad Q = q_1 + q_2, \tag{1}$$

where P is the price, q_i ($i = 1, 2$) is firm i's supply amount and a, b are parameters.

Based on the literature, subgame perfection is adopted as the equilibrium concept, and we consider a two-stage location-then-quantity game. We assume that the firms bear transport costs and they can set a supply amount for each location independently because arbitrage between consumers is assumed to be prohibitively costly. Further, the transport costs are the same for the firms and are linear to the supply amount. The unit transport cost for firm i is only dependent on the Euclidean distance d_i to a consumer and is quadratic with regard to the distance. Hence, the cost function is given by td_i^2, where t is a transport cost parameter and is assumed to be sufficiently low such that

$$a > 2t\left(l_x^2 + l_y^2\right) = 2t\left(l^2 + \frac{1}{l^2}\right), \tag{2}$$

which ensures that both firms serve the entire city, irrespective of the locations of the firms.[2] Let firm i's location be $(x_i, y_i) \in L$; then, the distance between a

[2] When the firms are diagonally located at the vertexes (e. g., $(x_1, y_1) = (l_x/2, l_y/2)$ and $(x_2, y_2) = (-l_x/2, -l_y/2)$), in which the distance between firms is maximized, a firm must serve at a location of the rival firm. This condition ensures such a positive supply with the

consumer at $(x, y) \in L$ and firm i is

$$d_i(x, y) = [(x_i - x)^2 + (y_i - y)^2]^{1/2}.$$

First, we analyze the second-stage game by backward induction. From (1), the local profit for firm i at point (x, y) is

$$\pi_i(x, y) = q_i(x, y)\left[P(x, y) - td_i(x, y)^2\right] \tag{3}$$

By solving the first-order conditions: $\partial\pi_i(x, y)/\partial q_i(x, y) = 0$, we have the equilibrium quantity for firm i at (x, y) as follows:

$$q_i(x, y) = \frac{a - 2td_i(x, y)^2 + td_j(x, y)^2}{3b} \quad \text{for } i, j \in \{1, 2\}, i \neq j. \tag{4}$$

Then, the equilibrium local profit is

$$\pi_i(x, y) = b\,[q_i(x, y)]^2 \quad \text{for } i, j \in \{1, 2\}, i \neq j,$$

where $q_i(x, y)$ is defined by (4). Hence, the total profit for firm i is given by

$$\Pi_i(x_i, y_i) = \iint_L \pi_i(x, y)dxdy. \tag{5}$$

equilibrium quantity in (4).

3 The result: location equilibrium

We analyze the first-stage game given the results in the second stage. We propose the three lemmas below.

Lemma 1 *The central agglomeration* ($x_1 = y_1 = x_2 = y_2 = 0$) *is a Nash equilibrium.*

Proof. We assume that firm 2 is located at the center ($x_2 = y_2 = 0$). Then, we have

$$\begin{aligned}
& \frac{27bl^2}{t}\left[\Pi_1(0,0) - \Pi_1(x_1, y_1)\right] \\
= \; & 12al^2\left(x_1^2 + y_1^2\right) - t\left[12l^2x^4 + x_1^2\left(1 + 5l^4 + 24l^2y_1^2\right) + y_1^2\left(5 + l^4 + 12l^2y_1^2\right)\right] \\
> \; & tx_1^2\left(23 + 19l^4 - 12l^2x_1^2 - 24l^2y_1^2\right) + ty_1^2\left(19 + 23l^4 - 12l^2y_1^2\right) \\
\geq \; & tx_1^2\left(23 + 19l^4 - 3l^4 - 6\right) + ty_1^2\left(19 + 23l^4 - 3\right) > 0
\end{aligned}$$

for all $x_1 \neq 0, y_1 \neq 0$, where the first inequality is due to (2) and the first inequality is due to $0 < x_1^2 \leq l^2/4$ and $0 < y_1^2 \leq 1/4l^2$. Hence, $x_1 = y_1 = 0$ is the unique best response to $x_2 = y_2 = 0$. Because of the symmetry with respect to the firms, it is clear that $x_2 = y_2 = 0$ is the unique best response to $x_1 = y_1 = 0$. ■

Lemma 2 *It cannot be a Nash equilibrium unless at least one firm locates on an axis.*

Proof. Without loss of generality, we assume that $0 < x_2 \leq l/2$, $0 < y_2 \leq 1/2l$ and $x_1^2 + y_1^2 \leq x_2^2 + y_2^2$. First, when $x_1 > 0$ or $y_1 > 0$, we have either $\Pi_1(-x_1, y_1) > \Pi_1(x_1, y_1)$ or $\Pi_1(x_1, -y_1) > \Pi_1(x_1, y_1)$ for all x_2, y_2. Second, when $x_1 < 0$ and $y_1 < 0$, we can show that $\Pi_1(0, y_1) > \Pi_1(x_1, y_1)$ for all x_2, y_2 if $y_1 \geq x_1/l^2$ and $\Pi_1(x_1, 0) > \Pi_1(x_1, y_1)$ for all x_2, y_2 if otherwise. Hence, the case of $x_1 \neq 0, y_1 \neq 0, x_2 \neq 0, y_2 \neq 0$ cannot be an equilibrium. ∎

Lemma 3 *It cannot be an equilibrium in which a firm locates on an axis except for the central agglomeration.*

Proof. We assume (without loss of generality) that firm 2 is on an axis ($x_2 = 0$ or $y_2 = 0$). First, when $x_2 = 0$, we have $\Pi_1(0, y_1) > \Pi_1(x_1, y_1)$ for all $x_1 \neq 0, y_1, y_2$. Second, when $y_2 = 0$, we obtain $\Pi_1(x_1, 0) > \Pi_1(x_1, y_1)$ for all $x_1, y_1 \neq 0, x_2$. These facts have shown that it cannot be an equilibrium unless both firms are located on the same axis. Next, we consider such cases.

Consider that $x_1 = x_2 = 0$. We assume (without loss of generality) that $|y_1| \leq |y_2|$. Then, we have $\Pi_1(0, 0) > \Pi_1(0, y_1)$ for all $y_1 \neq 0$, y_2. Thus, there is an incentive for a firm that is nearer to the center to move into the center, which shows the case that no firm is located at the center cannot be an equilibrium. Further, when a firm (without loss of generality, firm 2) is located at the center, we readily have $\Pi_1(0, 0) > \Pi_1(0, y_1)$ for all $y_1 \neq 0$. Hence, it cannot be an equilibrium unless both firms are located at the center when $x_1 = x_2 = 0$.

When $y_1 = y_2 = 0$, a similar calculation shows that it cannot be an equilibrium unless both firms are located at the center. Thus, we have the lemma.

■

Clearly, Lemma 1, Lemma 2 and Lemma 3 have established our main result as follows.

Proposition 1 *The central agglomeration is the unique Nash location equilibrium.*

4 Conclusion

We have shown that the central agglomeration result is as robust in a rectangular space as in a linear one or a circular one. This suggests that firms have a stronger incentive to reduce transport costs by establishing a central location that provides better access to consumers than to relax competition by locational dispersion in a spatial Cournot competition.

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