Discussion Paper Series

Central Agglomeration
of Monopolistically Competitive Firms[1]

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Abstract

This paper analyzes the location equilibrium for firms under monopolistic competition in a linear market à la Hotelling. The central agglomeration of firms is uniquely achieved with sufficiently low transport costs under three pricing policies: discriminatory, mill, and uniform delivered pricing. Welfare analysis shows that the agglomeration is also the first-best outcome. Moreover, the use of mill pricing by all firms is better for social surplus than is the use of any other pricing. Nevertheless, there is an incentive for the use of discriminatory pricing by each firm when all firms apply mill pricing.

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1 Introduction

Since Hotelling's (1929) landmark work, research involving spatial factors in competition models has shown considerable development. Hotelling's seminal result is that, as a consequence of location competition performed by two firms in a linear city, both firms assemble in the center. This phenomenon is termed as the "principle of minimum differentiation." It is a pioneering conclusion that as a result of reasonable decisions made by firms, economic activity concentrates geographically; this is an agglomeration phenomenon. Further, it should be noted that the works of Hotelling and his followers consider not just a mere geographical interpretation of space but also—among others—an interpretation of characteristic space and policy space (Downs, 1957), providing a vast source of insights.

On the other hand, d'Aspremont et al. (1979) have raised doubts on the appropriateness of this principle. This research has indicated that in Hotelling's (1929) similar linear city model, when considering location and price defined by spatial competition, as long as two firms are not located sufficiently apart from each other, price equilibrium does not exist. Further, the change to the quadratic function of linear transport costs, which is the original setting, led to the conclusion that the existence of the equilibrium price can be guaranteed, while at the same time, the two firms will mutually locate themselves apart from each other at the edges of the market.

In line with this research, Tabuchi (1994) and Irmen and Thisse (1998) have each analyzed the space's dimension expanded to 2 and m (≥ 2) dimensions. Consequently, they achieved surprising results: although the differentiation is maximized in one dimension, it does not occur in the other ($m-1$) dimensions. This indicates that with regard to product differentiation, if sufficient advantage over other firms is achieved in one aspect, it is already sufficient for all other aspects.

What must be taken into consideration concerning the abovementioned research is that there are alternative interpretations about space: geographical and product quality. For instance, regarding the Irmen and Thisse (1998) model, 2 of m dimensions are considered to be geographical, while it is rather difficult to regard the remaining ($m-2$) dimensions, as characteristic spaces. Nevertheless, models that can simultaneously accommodate both geographical and characteristic spaces—especially in terms of the agglomeration of firms handling differentiated goods—are more realistic. Hence, an analysis that clearly distinguishes geographical and characteristic spaces is required.

When considering linear cities such as those in Hotelling (1929) as geographical spaces, research such as Anderson and de Palma (1988) and De Fraja and Norman (1993, henceforth "DN") can be cited as including differentiation in terms of characteristic spaces within such

a framework. In both research, in cases where differentiation in terms of characteristic spaces occurs, it has been concluded that differentiation in terms of geographical spaces disappears (agglomeration in the center of the linear city). In comparison with the Irmen and Thisse (1998) model, it could be interpreted as multidimensional where differentiation of the geographical dimension does not occur but that of the characteristic dimension does take place.

Nevertheless, the research handle oligopolies (duopolies) and analyze a relatively small number of firms.[1] In this sense, this work performs an analysis with an increased number of firms: a monopolistic competition model. In this setting, the present work used the "OTT Model" monopolistic competition framework, which leads to a linear demand function, as developed by Ottaviano et al. (2002). (See Appendix A for a brief explanation in this regard.) Consequently, we have the benefit of performing a clear comparison with DN, which also uses a linear demand function. Further, in the monopolistic competition model, there is the additional advantage of allowing analytical solutions that would be difficult to achieve using the demand function with constant elasticity in the widely used Dixit and Stiglitz (1977) work. In compliance with these researches, this work treats the number of firms as a continuum (as opposed to discrete), and each firm is considered to be atomic. Therefore, although the behavior of each firm does not imply in a direct and strategic influence upon other firms, the spatial distribution, average price level.

In this two-stage game, all firms decide their location in the first stage and their price in the second stage (monopolistic competition). Consequently, the type of location-price equilibria determined by this two-stage game is analyzed. As a conclusion, based on the limitation of all goods being supplied to the entire linear city (sufficiently small transport costs), it is indicated that only the type characterized by agglomeration in the center can realize a location equilibrium. This is similar to the result achieved by Anderson and de Palma (1988) and DN. Here, we can interpete the case of sufficiently small transport costs as the case of sufficiently differentiated products becase these are inversely related.[2] As indicated by DN, *"Product differentiation, no matter whether it arises naturally or is chosen strategically, actively encourages the agglomeration of noncooperative oligopolists"* (p. 349). On the other hand, this is also valid for monopolistically competitive firms, indicating that if there is a constant differentiation in the characteristic space, there is no "differentiation" in the geographical space. This may be an expression of the highly differentiated and varied firm agglomeration phenomenon found in cities.

On the other hand, as another trend concerning spatial competition of homogeneous goods,

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[1] See Rydell (1967, 1971), Snyder (1971), and Greenhut et al. (1986) for the case of monopolies.

[2] Although the limitation of practical parameters is explained later (12), we can state that if transportation costs $T(1)$ are fixed, the differentiation is sufficiently great (c is the inverse index of differentiation).

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we should also refer to Cournot's quantity competition model. The pioneering researches Hamilton et al. (1989) and Anderson and Neven (1991) have included this quantity competition in the spatial competition model occurring in the linear space à la Hotelling. These works analyze a two-stage game comprised of an existing market where quantity competition takes place in each point of the linear space, as well as where each firm chooses one location and, while bearing transport costs, chooses the quantity to be supplied to each market. Consequently, Hamilton et al. (1989) achieved the "principle of minimum differentiation," where both firms agglomerate in the center of the linear space. Anderson and Neven (1991) generalized the number of firms (n firms), all firms choose the center of the linear space.[3]

This resulting agglomeration is also found in the present work. In addition, when considering linear demand, the hypothesis of firms supplying to the entire linear market is also similar to this work. The great difference between this work, DN, and other quantity competition models, and works such as d'Aspremont et al. (1979), is that in the latter, in the case of price competition with homogeneous goods, consumers only purchase goods from one firm. Hence, in price competition models with homogeneous goods, firms can effectively obtain a local monopolistic market. However, in the former models, irrespective of where other firms locate themselves, the market overlaps and local monopoly is not achieved. This process weakens the incentive for dispersion; in fact, it indicates that the center of the city, as the point where transport costs are most economical, is the most advantageous.

In the main model developed in this study, firms bearing transport costs are analyzed as practicing discriminatory pricing. As in DN, the most representative of these pricing policies— mill and uniform delivered pricing—are considered. We can confirm that when agglomeration in the center is considered as given, as in DN, discriminatory pricing maximizes firm profit, while mill pricing maximizes consumer surplus. We can also confirm that while social surplus is maximized with mill pricing, there are incentives for firms to choose discriminatory pricing. Thus, a voluntary choice of mill pricing, and the subsequent social surplus maximizing, is impossible.

DN indicates that, considering the sequential choice for a pricing strategy, when differentiation is small, the first mover choose mill pricing while the second mover select discriminatory pricing (asymmetric pricing policy). In this process, the first mover choose mill pricing and set it higher, appeasing the reaction from the second mover, thereby enabling the first mover to commit to mill pricing. This result is extremely interesting, but only possible in a duopoly. In a setting such as in this study, where there are various firms that cannot place a direct strategic effect on other firms, this commitment is impossible and discriminatory pricing must

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[3]These results are realized under the condition that as goods are supplied to every point in the market, transportation costs are sufficiently low.

be applied by firms.

Further, welfare analysis is performed. In the first-best analysis where price and location can be controlled, prices must be the unit transport costs (marginal cost), and agglomeration in the center must be achieved.

The remainder of this paper is organized as follows: in Section 2, the two-stage game for location-price decision is presented; in Section 3, price equilibrium is analyzed; in Section 4, location equilibrium is analyzed and the main results are presented; in Sections 5 and 6, the issues concerning pricing policies and welfare analysis are analyzed; Section 7 summarizes the results.

2   The Model

We consider the linear city expressed by the line segment $L = [0, 1]$, where consumers are uniformly and continuously distributed. The total population of consumers is normalized to one so that we obtain a density function of $f(x) = 1$ $(x \in L)$. They are identical (in tastes for exogenously given goods and income) except for their locations as described later in the demand function.

There is a continuum of firms of which the total mass is n, of which each firm only supplies one type of horizontally differentiated products.[4] Further, each firm is treated as infinitesimally small, and its action has no impacts on any other firm and the entire economy; that is, they engage in monopolistic competition à la Chamberlin. As in Dixit and J. Stiglitz (1977), the firms have the same cost conditions and the differentiation among their goods are symmetric; therefore, they can be effectively distinguished only by their locations. Hence, let the firm's index be expressed by a real number, $i \in [0, n]$.

We now hypothesize on the location-price two-stage game. In the first stage, all firms choose their locations simultaneously (location competition), and in the second stage, given the spatial distribution of firms, prices are simultaneously chosen (price competition). The spatial distribution of firms is expressed in a density function $g(x)$ such that

$$\int_0^1 g(x)dx = n.$$

Figure 1 illustrates an example of spatial distribution.

The demand q_i for good i for consumers with the same preferences is taken as in the following linear function.[5]

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[4] We regard n as a fixed number. Usually, this is endogenously decided by profits becoming zero by free entry in the market. Here, the interpretation is that, implicitly, the profits at n firm's equilibrium match fixed costs.

[5] Namely, $q_i = \max\{0, a - (b + cn)p_i + cnP\}$; however, later, it will be assumed as $q_i > 0$ unless otherwise specified.
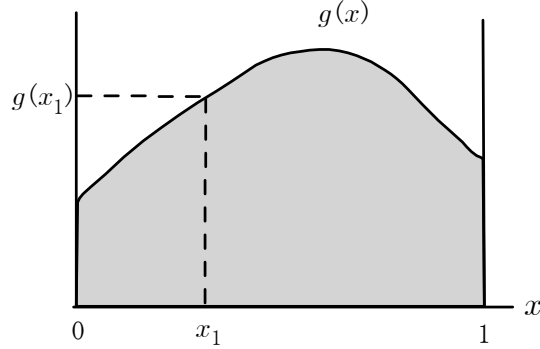
Figure 1: Example of spatial distribution: The mass of firms at x_1 is given by $g(x_1)$. The shaded area is equal to n.

$$q_i = a - (b + cn)p_i + cnP, \tag{1}$$

where p_i is the price of good i. a, b, c are parameters, and when $c \to 0$, the substitutability is low (the degree of differentiation is high), while when $c \to \infty$, perfect substitution (homogeneous products) is achieved. Hence, c is the indicator expressing the level of substitutability.

Further, the average price (price index) of all goods is given as

$$P = \frac{1}{n} \left(\int_0^n p_i di \right).$$

As part of profit-maximizing behavior, firms under monopolistic competition take the price index P as given. Now, this formulation is calculated as in Ottaviano et al. (2002), and a brief explanation on this can be found in Appendix A.

Each firm i exists in L and supplies goods to all consumers, earning profit. We assume that firms bear transport costs (shipping model) and can price discriminate for each consumer (market). Let the distance between a firm and a consumer be $d \geq 0$. Then, the transport costs function is linear to the supply amount, and the unit transport costs are only dependent on the distance and are expressed as $T(d)$[6]. This function monotonically increases in distance; further, it is convex and satisfies

$$T(0) = 0, T'(d) > 0, T''(d) \geq 0.$$

The simplest function satisfying this property is the linear function. If we consider $x_i \in L$ as the location of firm i and $y \in L$ as the location of the consumer, this function becomes

$$T(d) = td = t \, |x_i - y| \tag{2}$$

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[6] We have dropped index i from the function due to the symmetry ot firms. We will repeat this in profit functions.

($t > 0$ is the parameter expressing the transport costs per quantity, per distance). This linear function is assumed in many studies (DN, for example); however, this limitation is not necessary for the results in this paper.

The marginal cost necessary for the production of goods is constant and standardized to 0. In this case, the profit $\pi(x_i, y)$ earned from y by firm i located at x_i is written as

$$\pi(x_i, y) = [p(x_i, y) - T(|x_i - y|)] q(x_i, y), \tag{3}$$

where $p(x_i, y)$ is the price set by the firm at y, while $q(x_i, y)$ is the linear demand given in (1).

The profit earned from each point is totaled for the entire market $L = [0, 1]$ and expressed in the following, which represents firm i's total profit.

$$\Pi(x_i) = \int_0^1 \pi(x_i, y) dy \tag{4}$$

Resale between consumers are assumed to be prohibitively costly. In this case, as the relationship among markets is assumed as independent, firms can set prices so as to maximize their profits in each distinct point (market). Also, firms are symmetrical with regard to cost conditions; therefore, the firm's index i is accordingly abbreviated except where it is confusing to do so.

With regard to the solution concept, subgame perfection is used: the two-stage game is analyzed through backward induction. Hence, given the firm's spatial distribution, the two-stage game's price equilibrium is calculated in Section 3, and based on this, the location equilibrium of the first stage is analyzed in Section 4.

3 Price Equilibrium

Considering the firm's spatial distribution as given from the first stage, we search for the price equilibrium set by each firm. This spatial distribution is expressed as $g(x) \geq 0$ where

$$\int_0^1 g(x) dx = n.$$

We assumed that the firm's density function at x is shown.

Suppose that the firm i is located at x (index i is omitted), the profit at y given by (3) is maximized by the price $p(x, y)$. As a given price index, the first-order condition $(\partial \pi(x, y)/\partial p(x, y) = 0)$ is achieved through the following (Figure 2):

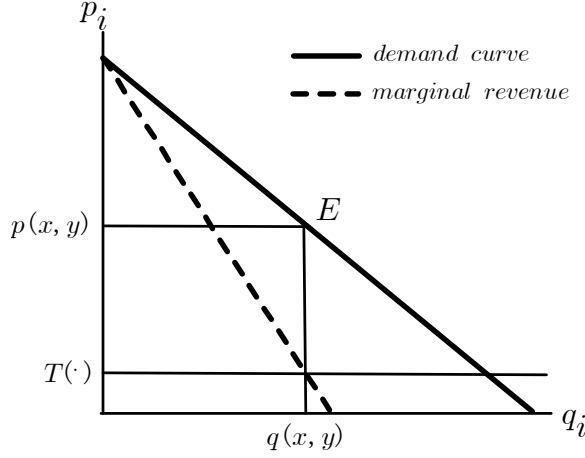$$p(x, y) = \frac{a + cnP(y)}{2(b + cn)} + \frac{T(|x - y|)}{2}, \tag{5}$$

Figure 2: Price equilibrium: point E shows equilibrium price and quantity supplied.

where $P(y)$ is the price index at y, and is given by the following:

$$P(y) = \frac{1}{n}\left(\int_0^1 p(x,y)g(x)dx\right).$$ (6)

Substituting (6) into (5) and summarizing, we have

$$P(y) = \frac{1}{2b+cn}\left[a + \frac{b+cn}{n}D(y)\right],$$ (7)

where

$$D(y) \equiv \int_0^1 T\left(|x-y|\right)g(x)dx$$ (8)

expresses the total transport costs for all firms to y. In other words, it is the total of transport costs from (consumers in) y to all firms, and the greater the relative concentration of firms at y, the lower is $D(y)$. For instance, when all firms are concentrated in point 0 $(g(0) = n; g(s) = 0$ otherwise), $D(0) = 0$ and $D(1) = n$, the value in point 0 close to the firms will become smaller, and that in point 1 distant from the firms will increase. Therefore, in markets where firms are agglomerated, the price index tends to decrease, and price relatively decreases (pro-competitive effect).

Substituting (7) into (5), price equilibrium $p^*(y)$ given as spatial distribution is analytically achieved as in the following (the superscript asterisk expresses equilibrium):

$$p^*(x,y) = \frac{2a + cD(y)}{2(2b+cn)} + \frac{T\left(|x-y|\right)}{2}.$$ (9)

Further, from the supply amount $q^*(x,y)$ in equilibrium obtained in (1), we can confirm the following:

$$q^*(x,y) = (b+cn)\left[p^*(x,y) - T\left(|x-y|\right)\right].$$ (10)

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Inserting the above in (3), we obtain

$$\pi^*(x, y) = [p^*(x, y) - T(|x - y|)] q^*(x, y) \tag{11}$$

$$= (b + cn) [p^*(x, y) - T(|x - y|)]^2 = (b + cn)^{-1} [q^*(x, y)]^2.$$

Hence, the profit obtained from each point is proportional to the square of the supply amount. The total profit in equilibrium is expressed in the following from (11):

$$\Pi^*(x) = \int_0^1 \pi^*(x, y) dy = (b + cn) \int_0^1 [p^*(x, y) - T(|x - y|)]^2 dy.$$

Moreover, to guarantee the service of the market for all the points, the greatest value for transport costs, $T(1)$, is limited by the following:[7]

$$T(1) < T_{\text{trade}} \equiv \frac{2a}{2b + cn}. \tag{12}$$

For the purpose of later analysis, the spatial distribution of full agglomeration is defined here.

Definition 1 *We call the spatial distribution $g(s)$ "full agglomeration" when there exists $k \in L$ such that*

$$g(x) = \begin{cases} n & \text{if } x = k \\ 0 & \text{otherwise} \end{cases}. \tag{13}$$

Otherwise, in all other cases, spatial distribution is called "non-full agglomeration." In this case, the support for $g(x)$ is not singleton, and in L, firms with positive density exist at least in two points.[8] Henceforth, we first consider a non-full agglomeration type of spatial distribution as an object of analysis, and we consider the case of full agglomeration in subsection 4.2.

4   Location Equilibrium

The location equilibrium in the first stage in the location game is defined as a situation where there is no incentive for a firm to change its location, when all other firms' locations are given. We refer to such a spatial distribution as the equilibrium spatial distribution. As shown later (in Appendix B), firms never locate themselves at the edges ($x = 0, 1$); therefore, all we have

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[7] In order to serve the market, $p^*(x, y) > T(|x - y|)$ must be satisfied with regard to all x, y. After adjusting this condition under $(x, y) = (0, 1)$ as well as $g(1) = n$; $g(s) = 0, s \neq 1$, we acquire the conditional equation (12). This condition states even if all other firms agglomerate at 1, and the firm locate itself at 0, which is the farthest location from 1, it serves the most competitive location 1.

[8] In this case, $D(y) > 0$ for all $y \in L$ can be guaranteed, which simplifies the analysis. On the other hand, in case of full agglomeration, clearly $D(y) = 0$ in agglomeration point (k), and $D(y) > 0, (y \neq k)$ for all other cases.

to focus on here is the inner solution. Since a firm's location, x^*, is the location equilibrium, it implies that the following equation is satisfied given $g(x)$:

$$\Pi^*(x^*) \geq \Pi^*(x) \quad \text{for } \forall x \in L. \tag{14}$$

Due to the symmetric properties of firms, the above equation must be equally satisfied for all firms. If there exists a firm that does not satisfy it, it means that the given $g(x)$ is not the equilibrium spatial distribution. Each firm is negligibly small and affects neither the other firms' profits nor the spatial distribution; thus, the equilibrium spatial distribution, $g^*(x)$, must have the following properties:

$$\begin{aligned} \Pi^*(x) &= \Pi \quad \text{if } g^*(x) > 0 \\ \Pi^*(x) &\leq \Pi \quad \text{if } g^*(x) = 0 \end{aligned} .$$

Here, $\Pi > 0$ (constant) represents the gross profits earned by each firm.[9] In other words, in an area where firms exist with positive density, the profits earned by each firm are the same, while in an area where no firms exist, if a firm located itself there, the firm would make smaller profits. From (14), a firm's optimal location, x^*, must satisfy the following conditions:

$$\left. \frac{\partial \Pi^*}{\partial x} \right|_{x=x^*} = 0, \left. \frac{\partial^2 \Pi^*}{\partial x^2} \right|_{x=x^*} \leq 0.$$

We will analyze the properties of the optimal location based on this condition below.

4.1 Some properties of location equilibrium

Let us consider the properties of location equilibrium in detail. We derive the following by partially differentiating the total profits, $\Pi^*(x)$, with respect to x:

$$\begin{aligned} \frac{\partial \Pi^*(x)}{\partial x} &= \frac{1}{b+cn} \left\{ \frac{\partial}{\partial x} \int_0^x [q(x,y)]^2 \, dy + \frac{\partial}{\partial x} \int_x^1 [q(x,y)]^2 \, dy \right\} \\ &= \Delta Q(x), \end{aligned} \tag{15}$$

where

$$\Delta Q(x) \equiv \int_x^1 q(x,y) T'\left(|x-y|\right) dy - \int_0^x q(x,y) T'\left(|x-y|\right) dy. \tag{16}$$

A firm's optimal location, x^*, satisfies $\Delta Q(x^*) = 0$ from the first-order condition, $\partial \Pi^*(x)/\partial x = 0$; thus, the following is satisfied:

$$\int_0^{x^*} q(x^*,y) T'\left(|x^*-y|\right) dy = \int_{x^*}^1 q(x^*,y) T'\left(|x^*-y|\right) dy \tag{17}$$

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[9] At long-run equilibrium with free entry and exit, Π must be zero. In this chapter, the fixed costs for each firm are implicitly assumed to be $K = \Pi$; then, the excess profits disappear.

In order to understand what this implies, we now consider a case where the transport costs are linear as indicated in equation (2). In this case of $T'(d) = t$, the first-order condition (17) can be reduced to

$$\int_0^{x^*} q(x^*, y)dy = \int_{x^*}^1 q(x^*, y)dy.$$

This implies that a firm's optimal location is the point where aggregate supply by the firm to the left of that point in the entire market (left-hand side) is equal to that by the firm to the right of that point (right-hand side). The marginal increase in profit earned by a firm making a small leftward movement must be balanced with the marginal decrease in profit at the optimal location. In a case where the transport costs are linearly proportional to distance, the change in aggregate supply is proportional to the change in distance.[10] From (11), profits are proportional to the square of the quantity supplied, and it is obvious that the marginal profits are proportional to the quantity supplied. Hence, the marginal increase in profit earned by a firm making a small leftward movement is aggregate supply to the left of the point in the entire market, while the marginal decrease in profit is aggregate supply to the right of the point. At the optimal location, the aggregate supply on the left-hand side must be equal to that on the right-hand side.

Now let us consider (17) without the assumption that the transport costs are proportional to distance. The optimal location should be the point at which the weighted quantity supplied to the left is equal to that supplied to the right, after weighting the degree of increase of the transport costs in relation with the increase in distance. In a case where such a degree is incremental, $T''(d) > 0$, the greater the distance between a firm and a market, the sharper is the increase in the transport costs; thus, the advantages in coming closer to and the disadvantages in growing apart from a market that is distant from a firm's location are greater than the respective advantages and disadvantages at a nearby market. The advantages of approaching and the disadvantages of distancing from farther markets are greater than those in the nearer markets. Therefore, firms must select their locations such that the differences between them are adjusted. In a case where the transport costs are linearly proportional to distance, when a firm relocates, the resulting changes in advantages and disadvantages at both a farther market and a nearer market are the same, and such a relocation will not favor either side. We obtain the following lemma by elaborating on this optimal location:

Lemma 1 *The optimal location, x^*, uniquely exists in $(0, 1)$ given by (17).*

Proof. See Appendix B.

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[10] We readily have $\partial q(x,y)/\partial x = \pm(b + cn)T'(|x - y|)/2$ if $x \lessgtr y$ from (9) and (10).

This optimal location is determined solely on the basis of firms' spatial distribution. Each firm is negligibly small, and the changes in its location do not affect the spatial distribution; therefore, the optimal location for all the firms is the same ($x_i^* = x^*$, for $\forall i$). Hence, it is clear that the spatial distribution of non-full agglomeration cannot be the equilibrium because when there are more than two supports of the spatial distribution, there exist firms that situate themselves at locations other than x^*, and they have an incentive to change their locations. We will address some properties of the optimal dispersion and then present a detailed analysis on agglomeration in subsection 4.2.

The optimal location, x^*, depends on the firms' spatial distribution and has the following properties (derivation in Appendix C):

Remark 1

$$0 < x^* < 1/2 \quad \text{if and only if} \int_0^{1/2} D(y)T'\left(|1/2 - y|\right) dy > \int_{1/2}^1 D(y)T'\left(|1/2 - y|\right) dy,$$

$$x^* = 1/2 \quad \text{if and only if} \int_0^{1/2} D(y)T'\left(|1/2 - y|\right) dy = \int_{1/2}^1 D(y)T'\left(|1/2 - y|\right) dy,$$

$$1/2 < x^* < 1 \quad \text{if and only if} \int_0^{1/2} D(y)T'\left(|1/2 - y|\right) dy < \int_{1/2}^1 D(y)T'\left(|1/2 - y|\right) dy.$$

For example, consider the first case when the following is satisfied:

$$\int_0^{1/2} D(y)T'\left(|1/2 - y|\right) dy > \int_{1/2}^1 D(y)T'\left(|1/2 - y|\right) dy. \tag{18}$$

$D(y)$, given by (8), is an inverse indicator of a firm's agglomeration and proximity with regard to the location, y, and indicates that the larger the value of $D(y)$, the smaller is the number of firms around the location. (18) indicates that there are relatively more firms located in the right half of a city, $[1/2, 1] \subset L$.

In this manner, that the optimal location depends on the firms' distribution comes from the substitutability between goods, that is, competition among firms. In a case of non-substitutability (i.e., a case where the degree of product differentiation is considerably large), each firm practically becomes monopolistic. Thus, in this model, $c = 0$, and we acquire the following:[11]

Corollary 1 *When c is zero (a case of non-substitutability), the optimal location for all the firms is the center of the city (1/2).*

Proof. See Appendix D.

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[11] In the other extreme case of perfect substitution, $c \to \infty$, we cannot analyze the equilibrium since T_{trade} becomes 0 in the equation of (12).

In other words, firms determine their locations such that their total transport costs are minimized. This is virtual monopoly. In a case of non-substitutability, firms determine their locations on the basis of the geographical element, i.e., the center of the market, rather than their competitors' distributions.

4.2 Full agglomeration

We limit the firms' spatial distribution, $g(x)$, to the function of (13) in Definition 1, that is, we focus on the case where all the firms fully agglomerate at location k. Assume that the location is the right half of a city, $k \in [1/2, 1] \subset L$, without loss of generality. From (8), the following is true:

$$D(y) = nT\left(|k - y|\right).$$

From (9), the equilibrium price at location y of a good produced by a firm at location x is given by:

$$p^f(x, y) = \frac{2a + cnT\left(|k - y|\right)}{2(2b + cn)} + \frac{T\left(|x - y|\right)}{2}. \tag{19}$$

Now, the superscript f represents "full agglomeration." From (11), the profits a firm earns at x are given by

$$\pi^f(x, y) = (b + cn)\left[p^f(x, y) - T\left(|x - y|\right)\right]^2. \tag{20}$$

Its total profits are given by

$$\Pi^f(x) = \int_0^1 \pi^f(x, y) dy. \tag{21}$$

For this full agglomeration to be an equilibrium, all we have to do is to check whether or not there exists an incentive for a firm to deviate from k. Here, we have the following proposition.

Proposition 1 *Given $T(1) < T_{\text{trade}}$, the central agglomeration is the unique equilibrium spatial distribution.*

Proof. See Appendix E.

This result is based on the following factors. First, the transport costs are convex with regard to distance. Due to such convexity, the greater the distance between a firm and a market, the greater is the drop in the quantity supplied. Considering the above reasons, there is a strong incentive for firms to select the center as their location. On the other hand, if the transport costs were concave with regard to distance, the greater the distance between a firm and a market, the lesser the drop in the quantity supplied; thus, it is possible that the central

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location might not be an equilibrium. Note that the proof of Proposition 1 is based on this assumption of convexity.

The second factor is the uniform distribution of consumers. For instance, in an extreme case where all consumers exist at location 0, all firms will also locate themselves at location 0.

5   Mill and Uniform Delivered Pricing

As DN analyze, the other pricing policies that firms employ in the spatial competition model include mill and uniform delivered pricing. In this section, we conduct the same analysis, except with different pricing policies. Here, we do not consider a case where one firm (group) and the other firm (group) employ different pricing policies, or a case of mixed strategy; our concern is a case where all the firms employ the same pricing policy. We will show that the central agglomeration is the unique equilibrium in the case of mill and uniform delivered pricing for low transport costs as we have shown in the discriminatory pricing case. We will compare the profits of the pricing policies and briefly analyze issues in selecting the policies.

5.1   Central agglomeration under mill and uniform delivered pricing

Mill pricing charges consumers the full transport costs. Thus, the full price for a consumer is the mill price plus the transport costs from the location of the firm to that of the consumer. Firms make the same profits per unit of supply irrespective of where they ship the product. In other words, the buying price faced by the consumer at y is given by $p^m(x) + T(|x - y|)$, where the mill price of a firm at x is $p^m(x)$. Then, the demand, $q^m(x, y)$ (wherein the superscript m represents the mill pricing, while the index i, which represents firms, is omitted), is given by :

$$q^m(x, y) = a - (b + cn)\left[p^m(x) + T(|x - y|)\right] + cnP(y). \tag{22}$$

Also, the total profits earned by a firm are given by

$$\Pi^m(x) = p^m(x) \int_0^1 q^m(x, y) dy. \tag{23}$$

On the other hand, the uniform delivered pricing is a policy where firms sell their products to consumers at the same price, regardless of their location. In other words, the demand of each consumer, q^u, is independent of his location and each firm's location, and it is given by the following:

$$q^u = a - (b + cn)p^u + cnP$$

where the uniform delivered price set by a firm located at x is p^u and the total profits earned by the firm are given by

$$\Pi^u(x) = q^u \int_0^1 \left[p^u - T\left(|x - y|\right) \right] dy \tag{24}$$

Firms set a price in the second stage so that their total profits are maximized, and taking that as given, they select their locations in the first stage. As in the case of discriminatory pricing, we assume that the transport costs are low, such that every consumer purchases a good, i.e.,

$$T(1) < T_{\text{trade2}} \equiv \frac{a}{2b + cn} \tag{25}$$

Note that since $T_{\text{trade2}} < T_{\text{trade}}$, (25) is more demanding than (12). Here, we have the same spatial equilibrium as follows.

Proposition 2 *Assume that (25) holds. Then, the central agglomeration is the unique spatial equilibrium under mill and uniform delivered pricing.*

Proof. See Appendix F.

5.2 Full prices and profits among three pricing policies

When the central agglomeration is achieved under each pricing policy, we compare the full price $p^j(y)$ for the consumer at y, where $j \in \{\text{disc}, \text{mill}, \text{uni}\}$ represents a pricing policy. Here, "disc" represents discriminatory pricing, while "mill" and "uni" are mill and uniform delivered pricing, respectively. The prices under these pricings are calculated from (19) with $k = x = 1/2$ and from Appendix F as follows[12]:

$$
\begin{aligned}
p^{\text{disc}}(y) &= \frac{1}{2b + cn} \left[a + (b + cn)T\left(\left| \frac{1}{2} - y \right| \right) \right] \\
p^{\text{mill}}(y) &= \frac{1}{2b + cn} \left(a - b\bar{T} \right) + T\left(\left| \frac{1}{2} - y \right| \right) \\
p^{\text{uni}}(y) &= \frac{1}{2b + cn} \left[a + (b + cn)\bar{T} \right],
\end{aligned}
\tag{26}
$$

where

$$\bar{T} \equiv \int_0^1 T\left(\left| \frac{1}{2} - y \right| \right) dy$$

is the aggregate transport costs from the center. In other words, this value can be interpreted as the average transport costs from the center for consumers in the city because the mass of

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[12] We can derive these prices from outputs in Appendix F as follows. Solving simultaneous equations (43) and $P(y) = p^m(1/2) + T(|1/2 - y|)$ with $x = 1/2$, we easily derive the mill price. Solving simultaneous equations (47) and $P = p^u$ with $x = 1/2$, we can derive the uniform delivered price.

consumers is unity. The following relationships hold from (26):

$$p^{\text{uni}}(y) - p^{\text{disc}}(y) = \frac{b + cn}{2b + cn}\left[\bar{T} - T\left(\left|\frac{1}{2} - y\right|\right)\right]$$

$$p^{\text{disc}}(y) - p^{\text{mill}}(y) = \frac{b}{2b + cn}\left[\bar{T} - T\left(\left|\frac{1}{2} - y\right|\right)\right]$$

$$p^{\text{uni}}(y) - p^{\text{mill}}(y) = \bar{T} - T\left(\left|\frac{1}{2} - y\right|\right).$$

Define $\bar{y}$ by the solution to[13]

$$T\left(\left|\frac{1}{2} - \bar{y}\right|\right) = \bar{T}.$$

Clearly, there exist two solutions of $\bar{y}$ which are symmetric to the center. We redefine $\bar{y} \in (0, 1/2)$ as the smaller solution of this equation. For example, suppose that the transport cost is linear, $T(d) = td$, then $\bar{y} = 1/4$. If the transport costs are given by a power function, $T(d) = td^\theta$ $(\theta \geq 1)$, then we easily derive $\bar{y} = \left[1 - (\theta + 1)^{-1/\theta}\right]/2$, which is decreasing in θ. In the limiting case of $\theta \to \infty$, we get $\bar{y} \to 0$.

We therefore obtain the relation between the prices in the three cases as given by the following expressions:

$$\begin{array}{ll}
p^{\text{mill}}(y) > p^{\text{disc}}(y) > p^{\text{uni}}(y) & \text{if} \quad y \in [0, \bar{y}) \cup (1 - \bar{y}, 1] \\
p^{\text{mill}}(y) = p^{\text{disc}}(y) = p^{\text{uni}}(y) & \text{if} \quad y = \bar{y}, 1 - \bar{y} \\
p^{\text{mill}}(y) < p^{\text{disc}}(y) < p^{\text{uni}}(y) & \text{if} \quad y \in (\bar{y}, 1 - \bar{y})
\end{array}.$$

Figure 3 illustrates these prices in the linear case, $T(d) = td$, with the parameters: $(t, a, b, c, n) = (1, 4, 1, 1, 1)$. This is the same result as DN (see Figure 1 on page 349 of their paper). Figure 4 shows the case of the quadratic transport cost, $T(d) = td^2$, with the same parameter values. In this case, $\bar{y} = (3 - \sqrt{3})/6 \approx 0.21$.

From (20) as well as (44) and (48) in Appendix F, the total profits, Π^j, under each pricing policy are given by

$$\Pi^{\text{disc}} = \frac{b + cn}{(2b + cn)^2} \times \int_0^1 [A(y)]^2 \, dy, \tag{27}$$

$$\Pi^{\text{uni}} = \Pi^{\text{mill}} = \frac{b + cn}{(2b + cn)^2}\left[\int_0^1 A(y) dy\right]^2 \tag{28}$$

where

$$A(y) \equiv a - bT\left(\left|\frac{1}{2} - y\right|\right). \tag{29}$$

Using the Cauchy-Schwarz inequality, we get[14]

$$\int_0^1 [A(y)]^2 \, dy > \left[\int_0^1 A(y) dy\right]^2$$

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[13] Clearly, $\bar{y}$ always exists inside the city due to the intermediate value theorem.

[14] Since $A(y)$ is not constant with respect to y, the strict inequality holds in the present case.
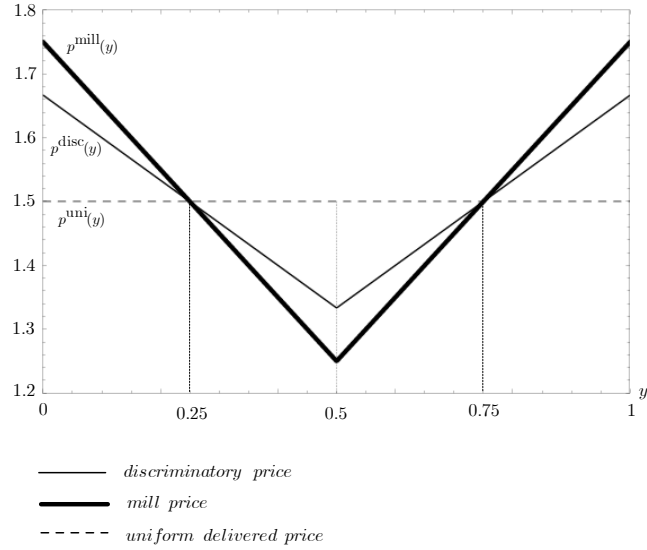
15

Figure 3: Full prices under three pricing policies with linear transport costs.
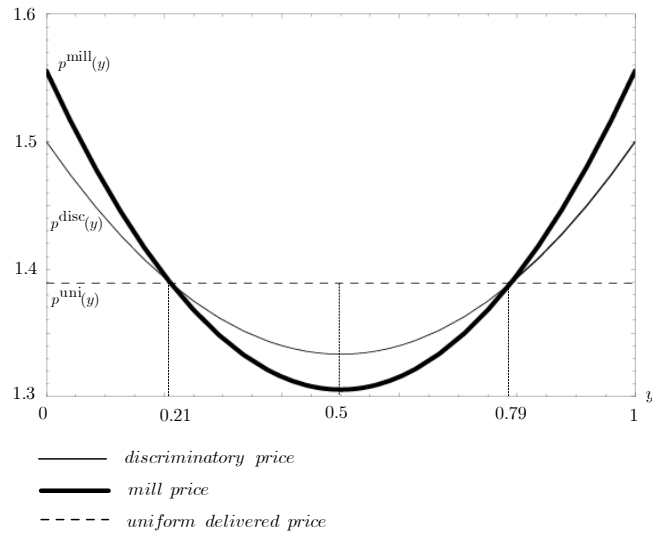

Figure 4: Full prices under three pricing policies with quadratic transport costs.

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Hence, we have

$$\Pi^{\text{disc}} > \Pi^{\text{uni}} = \Pi^{\text{mill}}$$

In other words, the discriminatory pricing is the most desirable for firms. This result is the same as that in the duopoly case (DN) as well as that in the monopoly case (Beckmann, 1970).

6 Welfare Analysis

We now consider social welfare by defining the social welfare function as the sum of producer surplus (PS) and consumer surplus (CS) (social surplus, $SS = PS + CS$). PS and CS are given by

$$PS \equiv \int_0^n \Pi_i di$$
$$CS \equiv \int_0^n CS_i di$$

Π_i represents the profits earned by firm i given by (4), while the following is consumer surplus from firm i's products, where $p_i(y)$ and $q_i(y)$ are the price and demand of product i faced by consumers at y, respectively:[15]

$$CS_i \equiv \int_0^1 \frac{1}{2} \left[\frac{a}{b} - p_i(y) \right] q_i(y) dy. \tag{30}$$

Figure 5 shows these surpluses in a diagram. Note that the "*perceived demand*" for a firm given by (1) and the "*true demand*" are different in this monopolistic competition model.

6.1 First-best policy

We now consider the first-best policy where a social planner can control the prices of products and locations. In this subsection, we assume that the transport costs increase monotonically and are convex. Obviously, the optimal price must be equal to the marginal cost; therefore, the price that a firm located at x should set for products sold at y, $p^o(x, y)$, is equal to the unit transport costs as follows:

$$p^o(x, y) = T\left(|x - y|\right) \tag{31}$$

where the superscript o means "optimum." Since no firms earn gross profits in this case, we have $SS = CS$. Hence, we will maximize the consumer surplus. Since each firm is identical, maximizing CS_i is equivalent to maximizing CS. Let

$$T_{\text{opt}} \equiv \frac{a(2b + cn)}{2b(b + cn)} > T_{\text{trade}} \tag{32}$$

be the threshold of transport costs. We obtain the following proposition by solving this problem:

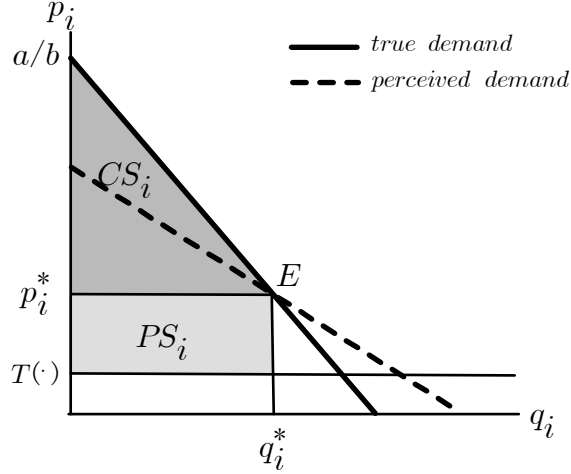[15] See Appendix G for the derivation of (30).

Figure 5: When the price and consumption are given by point E, then consumer surplus and producer surplus ($PS_i \equiv \Pi_i$) which are generated by the products i are represented by the shaded areas above. (The former is shown by the darker area.)

Proposition 3 *If $T(1) < T_{\mathrm{opt}}$ is satisfied, then the central agglomeration is the optimal spatial distribution.*

Proof. See Appendix H.

As in the case of the firms' optimal distributions, the optimal spatial distribution under the optimal price is the central agglomeration, where aggregate supply can be maximized (the total transport costs are minimized).[16]

6.2 Comparison among three pricing policies

We now compare the three pricing policies studied in the previous section in terms of welfare. We focus on the difference resulting from the pricing policies, assuming the spatial distribution is the central agglomeration in any case. PS^j of the pricing policy, $j \in \{\mathrm{disc}, \mathrm{mill}, \mathrm{uni}\}$, is $PS^j = n\Pi^j$ due to the symmetric property, so we can easily derive

$$PS^{\mathrm{disc}} > PS^{\mathrm{mill}} = PS^{\mathrm{uni}}$$

as in the previous section. Further, we can easily calculate CS and SS, using the equilibrium prices for the pricing policies obtained in the previous section. CS and SS under pricing policy j are CS^j and SS^j, respectively, and we have the following:

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[16]Proposition 3 also holds in the case of the second-best problem, in which the social planner can only control each firm's location, whereas its price is determined by a competitive equilibrium.

Proposition 4

$$CS^{\text{mill}} > CS^{\text{disc}} > CS^{\text{uni}}$$
$$SS^{\text{mill}} > SS^{\text{disc}} > SS^{\text{uni}} \quad .$$

Proof. See Appendix I.

This is the same result as in the monopoly case (Beckmann, 1976). Both CS and PS are the lowest in the uniform delivered pricing and are therefore the least attractive in terms of the social surplus. The discriminatory pricing enjoys the largest PS, while its CS is lower than that under the mill pricing. In terms of SS, mill pricing is better than discriminatory pricing. Therefore, mill pricing is the most desirable pricing policy. In a case where all firms employ mill pricing, however, there always exists an incentive for firms to change their pricing policy to discriminatory pricing.[17]

7 Concluding Remarks

In this paper, by applying the framework of monopolistic competition to the spatial competition model in a linear city, we have observed that the central agglomeration is supported as the location equilibrium. In relation to the spatial price competition model and the spatial quantity competition model with homogeneous goods, the findings in this paper conform to those of the latter. The outstanding characteristic of the spatial price competition model is that firms can enjoy "local monopoly" situations since consumers purchase products from only one of the firms. Therefore, they can make profits by specializing with their own customers, since they are located at a distance from their competitors. On the other hand, in the spatial quantity competition model, the market areas of each firm always overlap with those of other firms, and it is impossible for firms to acquire local monopoly through dispersion. Hence, firms must supply products to the entire city from the center and attempt to save transport costs.

Further, with regard to the roles of the characteristic space and the geographical space, we obtained the same results as those in the duopolistic model with product differentiation (Anderson and de Palma, 1988, and DN). That is, when goods are sufficiently differentiated in the characteristic space, they no longer try to differentiate in the geographical space. Differentiation is a crucial strategy to avoid a severe price competition. However, it also has a negative effect where firms move away to a less competitive market. The tendency for dispersion and agglomeration to coexist also conforms to the results of the analysis on homogeneous goods (Tabuchi, 1994, and Irmen and Thisse, 1998), namely, firms employ the differentiating policy in a spatial dimension, if at all.

In this paper, welfare losses do not appear in location decisions that are governed by

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[17]See Appendix J for the deviation of a firm.

competitive selection. In the homogeneous-product model, agglomeration results in losses in terms of transport costs, while in the monopolistic competition model where products are differentiated, agglomeration has desirable properties that eliminate such losses.

Finally, with regard to the pricing policies, we have observed the properties of mill and uniform delivered pricing along with discriminatory pricing. Discriminatory pricing is desirable for firms, but mill pricing is more desirable for consumers and the society. Firms do not select mill pricing on their own. Hence, enforcement of mill pricing by social planners will enhance the social surplus.

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APPENDIX

Appendix A: On "OTT model"

Here, we present a supplementary explanation on the derivation of the linear demand of (1). The formulation of our model is similar to that of the model in Ottaviano et al. (2002).

Each consumer has the same preference that is expressed by the utility function as follows:

$$U(q_A; q_i, i \in [0, n]) = \alpha \int_0^n q_i di - \frac{\beta - \gamma}{2} \int_0^n q_i^2 di - \frac{\gamma}{2} \left(\int_0^n q_i di \right)^2 + q_A, \tag{33}$$

where q_i is the quantity of goods i; q_A, the quantity of the numéraire good; and $\alpha > 0, \beta > \gamma > 0$, parameters. $\gamma \to 0$ represents no substitutes (very high degree of differentiation), whereas $\gamma \to \beta$ represents the perfect substitutes (homogeneous goods). Each consumer maximizes the utility under the budget constraint:

$$\int_0^n p_i q_i di + q_A = M, \tag{34}$$

where M is an exogenous income[18] and p_i is the price of good i.

Substituting (34) into (33), the first-order condition yields the following inverse demand function:

$$p_i = \alpha - (\beta - \gamma)q_i - \gamma \int_0^n q_i di. \tag{35}$$

Integrating this with respect to i yields the demand function

$$q_i = \frac{\alpha}{\beta + (n-1)\gamma} - \frac{1}{\beta - \gamma} p_i + \frac{\gamma}{(\beta - \gamma)[\beta + (n-1)\gamma]} \int_0^n p_i di.$$

Let the new parameters be

$$a \equiv \frac{\alpha}{\beta + (n-1)\gamma}, \quad b \equiv \frac{1}{\beta + (n-1)\gamma}, \quad c \equiv \frac{\gamma}{(\beta - \gamma)[\beta + (n-1)\gamma]},$$

$$P = \frac{1}{n} \left(\int_0^n p_i di \right).$$

Thus, the linear demand of (1) is obtained.

Appendix B: Proof of Lemma 1

Proof. From (16),

$$\frac{2(2b + cn)}{b + cn} \Delta Q(x) = \int_x^1 [2a + cD(y) - (2b + cn) T(|x - y|)] T'(|x - y|) dy$$

$$- \int_0^x [2a + cD(y) - (2b + cn) T(|x - y|)] T'(|x - y|) dy$$

$$= f_1(x) + f_2(x), \tag{36}$$

---

[18] The OTT model is a general equilibrium model with endogenous income.

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where

$$f_1(x) \equiv \frac{1}{2} \left[T(x) - T(1-x) \right] \left\{ -4a + (2b + cn) \left[T(x) + T(1-x) \right] \right\}, \tag{37}$$

$$f_2(x) \equiv c\Delta D(x), \tag{38}$$

where

$$\Delta D(x) \equiv \int_x^1 D(y) T' \left(|x - y| \right) dy - \int_0^x D(y) T' \left(|x - y| \right) dy. \tag{39}$$

Consider the properties of these functions. $f_1(x)$ is a decreasing in x since

$$\frac{\partial f_1(x)}{\partial x} = \left[-2a + (2b + cn) T(x) \right] T'(x) + \left[-2a + (2b + cn) T(1-x) \right] T'(1-x) < 0,$$

where the inequality holds because $T' > 0$; (12) yields

$$T(x), T(1-x) \le T(1) < \frac{2a}{2b + cn}.$$

Moreover,

$$f_1\left(\frac{1}{2} \right) = 0 \tag{40}$$

holds. From these relationships, $f_1(x)$ is a monotonically decreasing in x over the entire range $0 \le x \le 1$, and $f_1(x) = 0$ if and only if $x = 1/2$. Clearly,

$$f_1(0) > 0, \quad f_1(1) < 0$$

Next, consider $\Delta D(x)$.

$$\frac{\partial}{\partial x} \Delta D(x)$$
$$= -\int_x^1 D(y) T'' \left(|x - y| \right) dy - \int_0^x D(y) T'' \left(|x - y| \right) dy - 2D(x) T'(0) < 0 \tag{41}$$

holds, where the inequality is a result of $T' > 0, T'' \ge 0, D(x) > 0$. Moreover,

$$\Delta D(0) = \int_0^1 D(y) T' \left(|y| \right) dy > 0, \quad \Delta D(1) = -\int_0^1 D(y) T' \left(|1 - y| \right) dy < 0$$

are met. From these,

$$\frac{\partial \Delta Q(x)}{\partial x} < 0, \Delta Q(0) > 0, \Delta Q(1) < 0 \tag{42}$$

hold. Due to the continuity of $\Delta Q(x)$, x^* uniquely exists in $(0, 1)$ such that the first-order condition $\Delta Q(x^*) = 0$ is satisfied. Moreover, from (15) and (41), the second-order condition

$$\frac{\partial^2 \Pi^*(x)}{\partial x^2} = \frac{\partial}{\partial x} \Delta Q(x) < 0$$

is globally satisfied.[19]

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[19] Therefore, we can conclude that the corner solutions of $x^* = 0, 1$ do not exist.

Appendix C: Derivation of Remark 1

We utilize the results of Appendix B. Substituting $x = 1/2$ into (36) and (40) yields

$$\frac{2\left(2b + cn\right)}{b + cn}\Delta Q\left(\frac{1}{2}\right) = f_1\left(\frac{1}{2}\right) + f_2\left(\frac{1}{2}\right) = f_2\left(\frac{1}{2}\right) = c\Delta D\left(\frac{1}{2}\right).$$

Thus,

$$sgn\left[\Delta Q\left(\frac{1}{2}\right)\right] = sgn\left[\Delta D\left(\frac{1}{2}\right)\right]$$

holds. From (42) and the continuity of $\Delta Q(x)$,

$$\Delta Q\left(\frac{1}{2}\right) < 0 \Longleftrightarrow 0 < x^* < \frac{1}{2},$$

$$\Delta Q\left(\frac{1}{2}\right) = 0 \Longleftrightarrow x^* = \frac{1}{2},$$

$$\Delta Q\left(\frac{1}{2}\right) > 0 \Longleftrightarrow \frac{1}{2} < x^* < 1$$

are met, where $\Delta Q(x^*) = 0$.

Appendix D: Proof of Corollary 1

Proof. When $c \to 0$, from (38),

$$\lim_{c \to 0} f_2(x) = 0$$

holds. Since

$$-4a + (2b + cn)\left[T(x) + T(1 - x)\right] < 0$$

holds from (12), the first-order condition $\Delta Q(x) = 0$ is satisfied if and only if

$$T(x) - T(1 - x) = 0$$

from (37). Since $T' > 0$, this equation is satisfied only at $x = 1/2$.

Appendix E: Proof of Proposition 1

Proof. Let $1/2 < k \leqq 1$. From the total profit (21), we obtain

$$\left.\frac{\partial \Pi^f(x)}{\partial x}\right|_{x_i = k} = \frac{b + cn}{2\left(2b + cn\right)}\left[T(1 - k) - T(k)\right]\left\{2a - b\left[T(k) + T(1 - k)\right]\right\}.$$

From (12),

$$2a - b\left[T(k) + T(1 - k)\right] > 0.$$

Since $T' > 0$, when $1/2 < k \leqq 1$,

$$T(1 - k) - T(k) < 0.$$

Thus, each firm has an incentive to move toward the center. Hence, the full agglomeration at $1/2 < k \leqq 1$ cannot be the equilibrium spatial distribution.

Next, let $k = 1/2$. In this case, we obtain

$$\frac{2(2b + cn)}{b + cn} \times \frac{\partial}{\partial x} \Pi^f(x)\Big|_{k=1/2} = f_3(x) + f_4(x),$$

where

$$f_3(x) \equiv \frac{1}{2}[T(x) - T(1-x)]\{-4a + (2b + cn)[T(x) + T(1-x)]\},$$
$$f_4(x) \equiv cn\Delta D^f(x),$$

where

$$\Delta D^f(x) \equiv \int_x^1 T\left(\left|y - \frac{1}{2}\right|\right) T'(|x - y|)\, dy$$
$$- \int_0^x T\left(\left|y - \frac{1}{2}\right|\right) T'(|x - y|)\, dy$$

$f_3(x)$ is the same expression as $f_1(x)$ in (37) in Appendix B, and has the same property. This implies that $f_3(x)$ is also a monotonically decreasing in x over the interval $0 \leq x \leq 1$, and $f_3(x) = 0$ if and only if $x = 1/2$.

Moreover, $f_4(x)$ is similar to $f_2(x)$ in (38), and

$$f_4\left(\frac{1}{2}\right) = 0, \quad f_4(0) = cn\int_0^1 T\left(\left|y - \frac{1}{2}\right|\right) T'(|y|)\, dy > 0,$$
$$f_4(1) = -cn\int_0^1 T\left(\left|y - \frac{1}{2}\right|\right) T'(|1 - y|)\, dy < 0$$

hold. We also obtain

$$\frac{\partial \Delta D^f(x)}{\partial x} = -\int_x^1 T\left(\left|y - \frac{1}{2}\right|\right) T''(|x - y|)\, dy - \int_0^x T\left(\left|y - \frac{1}{2}\right|\right) T''(|x - y|)\, dy$$
$$- 2T\left(\left|x - \frac{1}{2}\right|\right) T'(0) \leq 0.$$

Then, $f_4(x)$ is a monotonically non-increasing in x. (The equality is satisfied if and only if $T'' = 0$ and $x = 1/2$.)

From these results, the profit function at $k = 1/2$ ($\Pi^f(x)\big|_{k=1/2}$) is maximized if and only if $x = 1/2$. Hence, each firm has no incentive to change its location when all the firms are located at the center. In other words, central agglomeration is the only location equilibrium.

Appendix F: Proof of Proposition 2

Proof. We divide the proof into two cases: (i) mill pricing and (ii) uniform delivered pricing.

(i) Mill pricing case.

First, let us consider the price equilibrium in the second-stage game. We temporarily assume that each firm serves the entire market; in other words, $q^m(x, y) > 0$ holds for all x and y. From (23), the first-order condition $\partial \Pi^m / \partial p^m = 0$ yields the equilibrium price as follows:

$$p^m(x) = \frac{1}{2(b+cn)} \int_0^1 [a + cnP(y) - (b+cn)T(|x-y|)]\, dy. \qquad (43)$$

Note that the term in the brackets is always positive under the assumption that $q^m(x, y) > 0$ for all x and y.

Next, we will proceed to the optimal location for the firm in the first-stage game. Substituting (43) into (23), we obtain the profit function as follows:

$$\Pi^m(x) = \frac{1}{4(b+cn)} \left\{ \int_0^1 [a + cnP(y) - (b+cn)T(|x-y|)]\, dy \right\}^2 \qquad (44)$$

Partially differentiating it with respect to x yields

$$\frac{\partial \Pi^m(x)}{\partial x} = \frac{1}{2} \left\{ \int_0^1 [a + cnP(y) - (b+cn)T(|x-y|)]\, dy \right\} [T(1-x) - T(x)].$$

From this, we can readily obtain

$$\left. \frac{\partial \Pi^m}{\partial x} \right|_{x=1/2} = 0,$$

and

$$\frac{\partial \Pi^m}{\partial x} \gtreqless 0 \quad \text{if} \quad x \lesseqgtr \frac{1}{2}.$$

Hence, the optimal location for the firm is $x = 1/2$. This is true for the other firms as well because they are mutually symmetric and the decision of one firm does not influence the decision of another firm.

Before concluding that central agglomeration is the unique equilibrium, we characterize a sufficient condition for positive supply in any market. The most difficult situation under which a firm will supply a positive amount of goods is one in which all the other firms are located at the endpoint of the city; in such a case, the firm would be located at the other endpoint and would be supplying its goods to the market of the agglomerated endpoint. For example, when all the firms are located at location 1, even if a deviating firm is located at 0, it must serve the most distant and competitive market at 1.

Let the mill price of each firm located at 1 be p_1^m; then, from (43) and $x = 1$, the following equations must hold:

$$
\begin{aligned}
P(y) &= p_1^m + T(1-y), \\
p_1^m &= \frac{1}{2(b+cn)} \int_0^1 [a + cnP(y) - (b+cn)T(1-y)]\, dy.
\end{aligned}
$$

Solving these simultaneously, we have

$$p_1^m = \frac{1}{2b+cn}\left[a - b\int_0^1 T\left(1-y\right)dy\right], P(y) = \frac{1}{2b+cn}\left[a - b\int_0^1 T\left(1-y\right)dy\right] + T\left(1-y\right). \tag{45}$$

Suppose that a deviating firm changes its location to location 0. From (43) and $x = 0$, the deviating firm sets its mill price p_0^m as

$$
\begin{aligned}
p_0^m &= \frac{1}{2\left(b+cn\right)}\int_0^1 \left\{a + cn\left[p_1^m + T\left(1-y\right)\right] - \left(b+cn\right)T\left(y\right)\right\}dy \\
&= \frac{1}{2\left(b+cn\right)}\left[a + cnp_1^m - b\int_0^1 T\left(y\right)dy\right], \tag{46}
\end{aligned}
$$

where p_1^m is given as in (45). Substituting (45) and (46) into (22), the demand of a consumer at 1 for the goods of the deviating firm is given as follows:

$$q^m\left(0,1\right) = \frac{1}{2b+cn}\left\{b^2\int_0^1 T\left(y\right)dy + \left(b+cn\right)\left[a - \left(2b+cn\right)T(1)\right]\right\}$$

Since $T'\left(y\right) > 0$ and $T''\left(y\right) \geq 0$,

$$\int_0^1 T\left(y\right)dy > 0$$

holds. Therefore, $q^m\left(0,1\right) > 0$ holds if

$$T(1) \leq \frac{a}{2b+cn}$$

is satisfied. Under this condition, the central agglomeration is clearly the unique location equilibrium.

(ii) Uniform delivered pricing case.

First, let us consider the price equilibrium in the second-stage game. Again, we temporarily assume that each firm serves the entire market; in other words, $q^u > 0$ holds for all the locations of all consumers and firms. From (24), the first-order condition $\partial\Pi^u/\partial p^u = 0$ yields the equilibrium price

$$p^u = \frac{1}{2}\left[\frac{a+cnP}{b+cn} + \int_0^1 T\left(|x-y|\right)dy\right]. \tag{47}$$

Next, we proceed to the optimal location of a firm in the first-stage game. Substituting (47) into (24), the profit function is obtained as follows:

$$\Pi^u(x) = \frac{1}{4\left(b+cn\right)}\left[a + cnP - \left(b+cn\right)\int_0^1 T\left(|x-y|\right)dy\right]^2 \tag{48}$$

Note that the term in the brackets is always positive under the assumption that $q^u > 0$.

Partially differentiating it with respect to x yields

$$\frac{\partial\Pi^u(x)}{\partial x} = \frac{1}{2}\left[a + cnP - \left(b+cn\right)\int_0^1 T\left(|x-y|\right)dy\right]\left[T(1-x) - T(x)\right].$$

From this, we can readily obtain

$$\left.\frac{\partial \Pi^u}{\partial x}\right|_{x=1/2} = 0,$$

and

$$\frac{\partial \Pi^u}{\partial x} \gtreqless 0 \quad \text{if} \quad x \lesseqgtr \frac{1}{2}.$$

Hence, the optimal location of the firm is $x = 1/2$. This is true for the other firms as well because they are symmetric with each other and the decision of one firm does not influence the decision of any other firm.

Similar to the case of mill pricing, we require the sufficient condition that each firm serves the entire market. Since $P = p^u$ under the uniform delivered pricing, from (47), we can readily obtain

$$P = p^u = \frac{1}{2b + cn} \left[a + (b + cn) \int_0^1 T\left(|x - y|\right) dy \right].$$

Since $T'\left(\cdot\right) > 0$ and $T''\left(\cdot\right) \geq 0$,

$$\int_0^1 T\left(|x - y|\right) dy > 0$$

holds. Hence, the following condition is met:

$$p^u > \frac{a}{2b + cn}$$

$q^u > 0$ always holds if $p^u > T(1)$. Therefore, the sufficient condition to be obtained is as follows:

$$T(1) \leq \frac{a}{2b + cn}$$

Under this condition, the central agglomeration is clearly the unique location equilibrium.

Appendix G: On consumer surplus

Here, we show that (30) can be interpreted as consumer surplus resulting from goods i. Appendix A already shows our utility function and the demand system of the OTT model. Those results can be used mutatis mutandis.

Notice that $\alpha = a/b$. By substituting the inverse demand function in (35) into the expression

$$\frac{1}{2} \left[\frac{a}{b} - p(i; y) \right]$$

in (30), this expression can be rewritten as

$$\frac{1}{2} \left\{ \frac{a}{b} - [\alpha - (\beta - \gamma)q(i; y) - \gamma Q] \right\} = \frac{1}{2} \left[(\beta - \gamma)q(i; y) + \gamma Q \right], \tag{49}$$

where

$$Q \equiv \int_0^n q(i; y) di.$$

29

By substituting (34) into (33), the representative consumer's utility V is rewritten by

$$V = \int_0^n V(i; y)di + M,$$

where

$$V(i; y) \equiv q(i; y) \left[\alpha - \frac{\gamma}{2}Q - \frac{\beta - \gamma}{2}q(i; y) - p(i; y) \right].$$

Substituting (35) into the above expression in the square bracket yields

$$\alpha - \frac{\gamma}{2}Q - \frac{\beta - \gamma}{2}q(i; y) - p(i; y)$$

$$= \alpha - \frac{\gamma}{2}Q - \frac{\beta - \gamma}{2}q(i; y) - [\alpha - (\beta - \gamma)q(i; y) - \gamma Q]$$

$$= \frac{1}{2} [(\beta - \gamma)q(i; y) + \gamma Q].$$

By comparing this with (49), we can confirm that

$$V(i; y) = \frac{1}{2} \left[\frac{a}{b} - p(i; y) \right] q(i; y).$$

Hence, (30) is the sum of (indirect) utilities obtained from goods i. The summation can be allowed since the utility function is quasi-linear, which implies transferable utility. CS represents the sum of the utilities of each good. Thus, CS is an appropriate measure for consumer welfare. Note that this expression does not include income M. We ignore M since it is exogenous and constant, and hence has no crucial impact on the analysis.

Appendix H: Proof of Proposition 3

Proof. Assuming first that the optimal spatial distribution is of non-full agglomeration type, we shall then show that this is a contradiction.

Substituting (31) into (30), the consumer surplus obtained from the goods of the firm located at x is rewritten as

$$CS_i = \int_0^1 \frac{1}{2} \left[\frac{a}{b} - T(|x - y|) \right] [a - (b + cn)T(|x - y|) + cnP(y)] \, dy.$$

Note that

$$P(y) = \frac{1}{n} \left(\int_0^1 p^o(x, y)g(x)dx \right) = \frac{1}{n} \left(\int_0^1 T(|x - y|)g(x)dx \right) = \frac{1}{n}D(y).$$

The first-order condition for the maximization of CS_i requires x_i to satisfy

$$2b\frac{\partial CS_i}{\partial x} = f_5(x) + f_6(x) = 0,$$

where

$$f_5(x) \equiv [T(x) - T(1-x)]\{-a(2b+cn) + b(b+cn)[T(x) + T(1-x)]\},$$

$$f_6(x) \equiv bc\Delta D(x),$$

and $\Delta D(x)$ is given by (39). We apply a similar calculation as that used in Appendix B; then, we can show that $f_5(x)$ and $f_6(x)$ are monotonically decreasing functions when (32) holds, and

$$f_5(0) > 0, \quad f_5\left(\frac{1}{2}\right) = 0, \quad f_5(1) < 0, \tag{50}$$
$$f_6(0) > 0, \quad f_6(1) < 0.$$

Hence, there uniquely exists an optimal location $x^o \in [0, 1]$ that satisfies the first-order condition $\partial CS_i/\partial x = 0$. Since each firm is negligibly small, the movement of a single firm has no impact on its spatial distribution function, the above calculation is valid for the other firms as well. This implies that an optimal location is common for all firms. Thus, a non-full agglomeration distribution cannot be optimal.

Next, we consider full agglomeration. Without any loss of generality, let us suppose that the agglomeration point is $k \in [1/2, 1]$. Then, an evaluation of $f_6(x)$ at $x = k$ yields

$$f_6(k) = \frac{1}{2}bcn\left\{[T(1-k)]^2 - [T(k)]^2\right\}.$$

Hence, $f_6(k) < 0$ holds when $1/2 < k \leq 1$. Moreover, from (50), $f_5(k) < 0$ also holds when $1/2 < k \leq 1$. Thus, the agglomeration at $k \in (1/2, 1]$ cannot be optimal since $1/2 < k \leq 1 \implies \partial CS_i/\partial x < 0$ holds.

Finally, the agglomeration at $k = 1/2$ (central agglomeration) is considered. A similar calculation yields

$$f_6\left(\frac{1}{2}\right) = 0,$$
$$\frac{\partial f_6(x)}{\partial x} = -\int_x^1 T\left(\left|y - \frac{1}{2}\right|\right) T''(|x-y|)\, dy - \int_0^x T\left(\left|y - \frac{1}{2}\right|\right) T''(|x-y|)\, dy$$
$$\quad - 2T\left(\left|x - \frac{1}{2}\right|\right) T\prime(0)$$
$$\leq 0,$$

where the equality holds only when $T''(\cdot) = 0$ and $x = 1/2$. Hence, when $k = 1/2$, the optimal location is $x^o = 1/2$. This implies that no rearrangements are required for an increase in social surplus. We can conclude that the central agglomeration is optimal.

Appendix I: Consumer Surplus of each pricing policy

A tedious calculation yields

$$CS^{\text{disc}} = \frac{n(b+cn)^2}{2b\,(2b+cn)^2} \int_0^1 [A(y)]^2\, dy, \tag{51}$$

$$CS^{\text{mill}} = \frac{n}{2b\,(2b+cn)^2} \int_0^1 \left\{ (b+cn)^2\, [A(y)]^2 + b^2 \left[\bar{T} - T\left(\left| y - \frac{1}{2} \right| \right) \right] \right\}^2 dy,$$

$$CS^{\text{uni}} = \frac{n(b+cn)^2}{2b\,(2b+cn)^2} \left[\int_0^1 A(y)dy \right]^2.$$

where $A(y)$ is defined in (29).

We can readily establish the following by Cauchy-Schwarz inequality:

$$CS^{\text{mill}} - CS^{\text{disc}} = \frac{b^2 n(3b+2cn)}{2\,(2b+cn)^2} \left\{ \int_0^1 T\left(\left| y - \frac{1}{2} \right| \right)^2 dy - \left[\int_0^1 T\left(\left| y - \frac{1}{2} \right| \right) dy \right]^2 \right\} > 0,$$

$$CS^{\text{disc}} - CS^{\text{uni}} = \frac{n(b+cn)^2}{2b\,(2b+cn)^2} \left\{ \int_0^1 [A(y)]^2\, dy - \left[\int_0^1 A(y)dy \right]^2 \right\} > 0.$$

Note that these strict inequalities arise because $T\left(|y - 1/2| \right)$ and $A(y)$ are not constant with respect to y.

Furthermore, from (27) and (51), we easily obtain

$$SS^{\text{mill}} - SS^{\text{disc}} = \frac{b^3 n}{2\,(2b+cn)^2} \left\{ \int_0^1 T\left(\left| y - \frac{1}{2} \right| \right)^2 dy - \left[\int_0^1 T\left(\left| y - \frac{1}{2} \right| \right) dy \right]^2 \right\} > 0,$$

$$SS^{\text{disc}} - SS^{\text{uni}} = \frac{n\,(b+cn)\,(3b+cn)}{2b\,(2b+cn)^3} \left\{ \int_0^1 [A(y)]^2\, dy - \left[\int_0^1 A(y)dy \right]^2 \right\} > 0,$$

where the inequalities hold due to the Cauchy-Schwarz inequality.

Appendix J: Deviation of firms from mill pricing to discriminatory pricing

Let us suppose that all the firms are located at the center. Then, we apply the mill pricing. From (26), the full price and price index for the consumer at y, $p^{\text{mill}}(y)$, and $P^{\text{mill}}(y)$ are the same and are given by

$$p^{\text{mill}}(y) = P^{\text{mill}}(y) = \frac{1}{2b+cn}\left(a - b\bar{T} \right) + T\left(\left| \frac{1}{2} - y \right| \right).$$

Similar to the manner of the analysis in our model in Section 2, let us suppose that the locations of the firms are fixed (central agglomeration) and that only the pricing policy is changed to discriminatory pricing. Then, from (5), the discriminatory price $\tilde{p}(y)$ for this firm is given by

$$\tilde{p}(y) = \frac{1}{2\,(b+cn)}\left[a + cnP^{\text{mill}}(y) \right] + \frac{1}{2}T\left(\left| \frac{1}{2} - y \right| \right).$$

Let $\tilde{\Pi}$ denote the total profit for this deviating firm. Then, we have

$$\tilde{\Pi} = \frac{1}{b+cn} \int_0^1 [\tilde{q}(y)]^2 \, dy,$$

where

$$\tilde{q}(y) = \frac{1}{2(2b+cn)} \left\{ 2(b+cn)\left(a - b\bar{T}\right) + bcn \left[T\left(\left| \frac{1}{2} - y \right| \right) - \bar{T} \right] \right\}$$

is the equilibrium demand of this firm at y. A tedious calculation yields

$$\tilde{\Pi} - \Pi^{\text{mill}} = \frac{b^2}{4(b+cn)} \left\{ \int_0^1 T\left(\left| y - \frac{1}{2} \right| \right)^2 dy - \left[\int_0^1 T\left(\left| y - \frac{1}{2} \right| \right) dy \right]^2 \right\} > 0,$$

where Π^{mill} is the profit for maintaining mill pricing, which is a result of (28). Thus, each firm can earn a higher profit by applying the discriminatory pricing instead of the mill pricing. This suggests that the case in which all the firms apply the mill pricing cannot be considered an equilibrium state when the game of simultaneously choosing pricing policies before the price game is considered for the firms.